Simulating Groundwater Flow and Solute Transport in A Karst Aquifer With Conduits

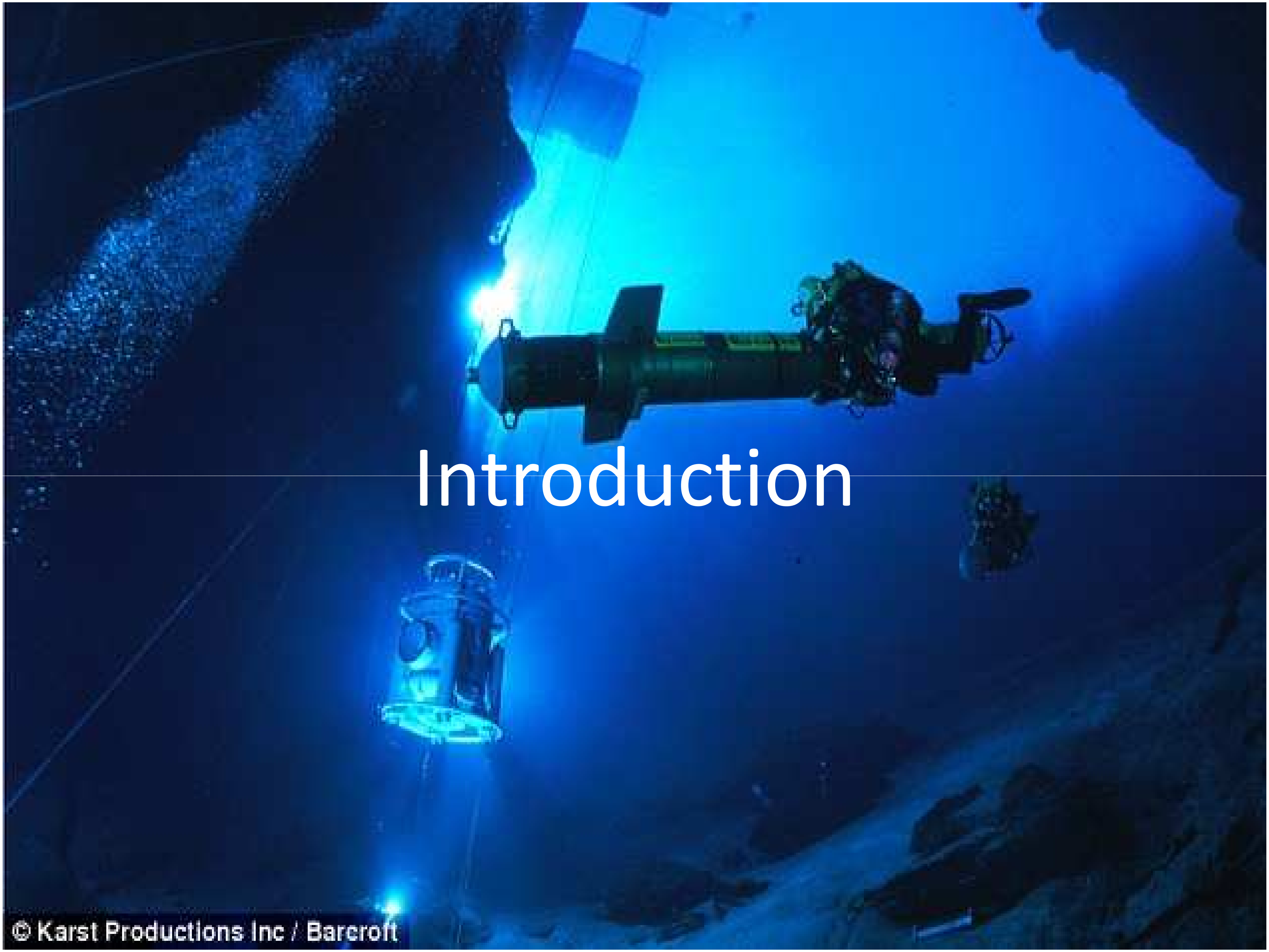
Bill X. Hu

Department of Earth, Ocean and Atmospheric Sciences
Florida State University

Outline of Presentation

- Introduction
- Purpose
- An evaluation of MODFLOW-CFP
- at the Laboratory Scale
- at the Sub-Regional Scale
- Stokes-Darcy Model
- Conclusions

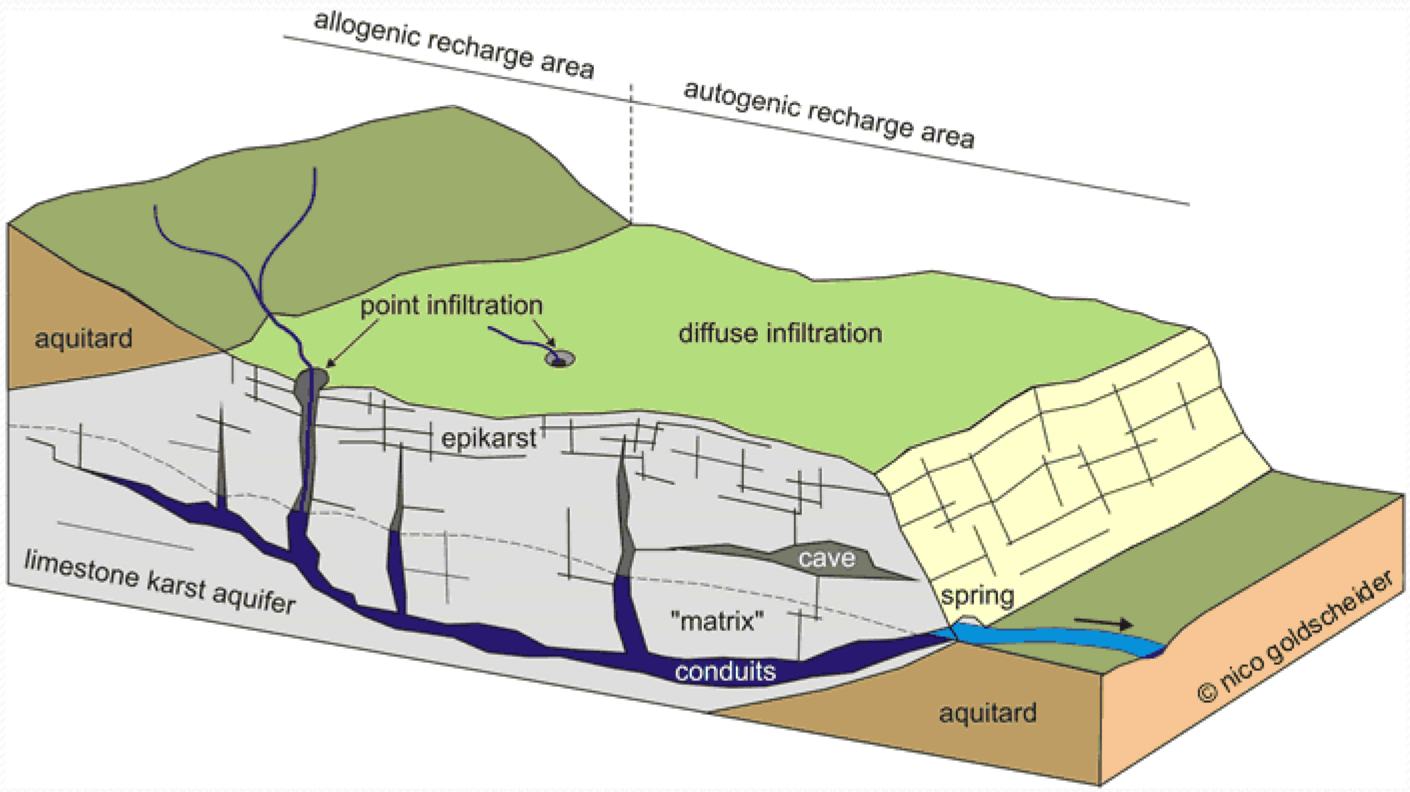
Introduction



Karst Aquifers

- Karst Aquifers are carbonate (e.g. limestone) aquifers with caves and conduit networks formed by dissolution.

Dissolution is primarily caused by carbonic acid – a product of the interaction between atmospheric CO_2 & water



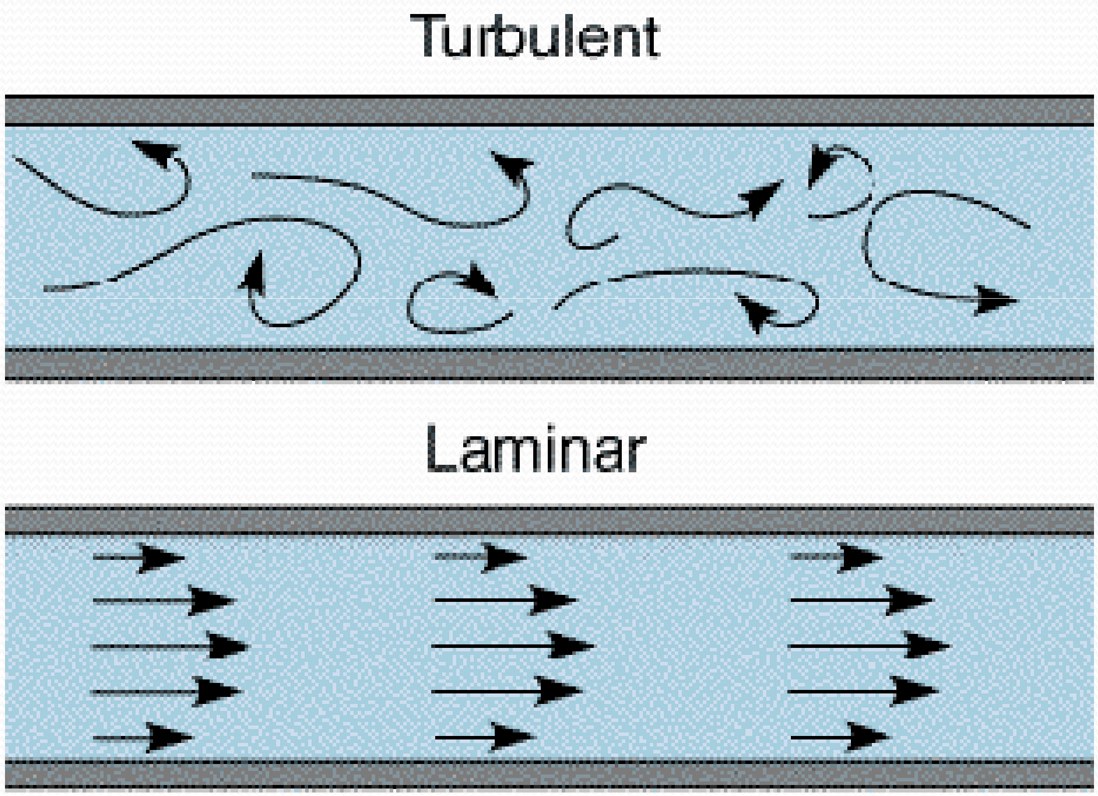
Karst aquifers

- Karst Aquifers are also known as dual porosity aquifers
- They have two porosity domains:
  - Matrix: composed of primary porosity
  - Conduit Network: composed of secondary porosity

Water Flow Types within Aquifers

- Turbulent flow is characterized by water flowing in non-parallel stream lines that form complex eddies

- Laminar flow is characterized by water flowing in parallel streamlines



Darcy's Law

- Many groundwater models assume that Darcy's Law governs/describes flow (e.g. MODFLOW-2005)

$$q = -K \frac{dh}{dl}$$

•In general this is true
•However there are some limitations, especially when applied to karst aquifers

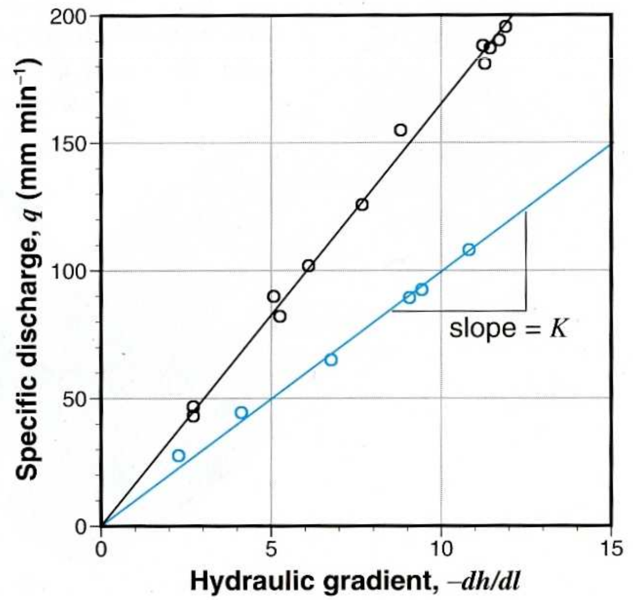


Figure 6.4 Darcy's (1856) original data showing a linear relationship between specific discharge and hydraulic gradient for two different sands.

From Hornberger et al., 1998

Limitations of Darcy's Law

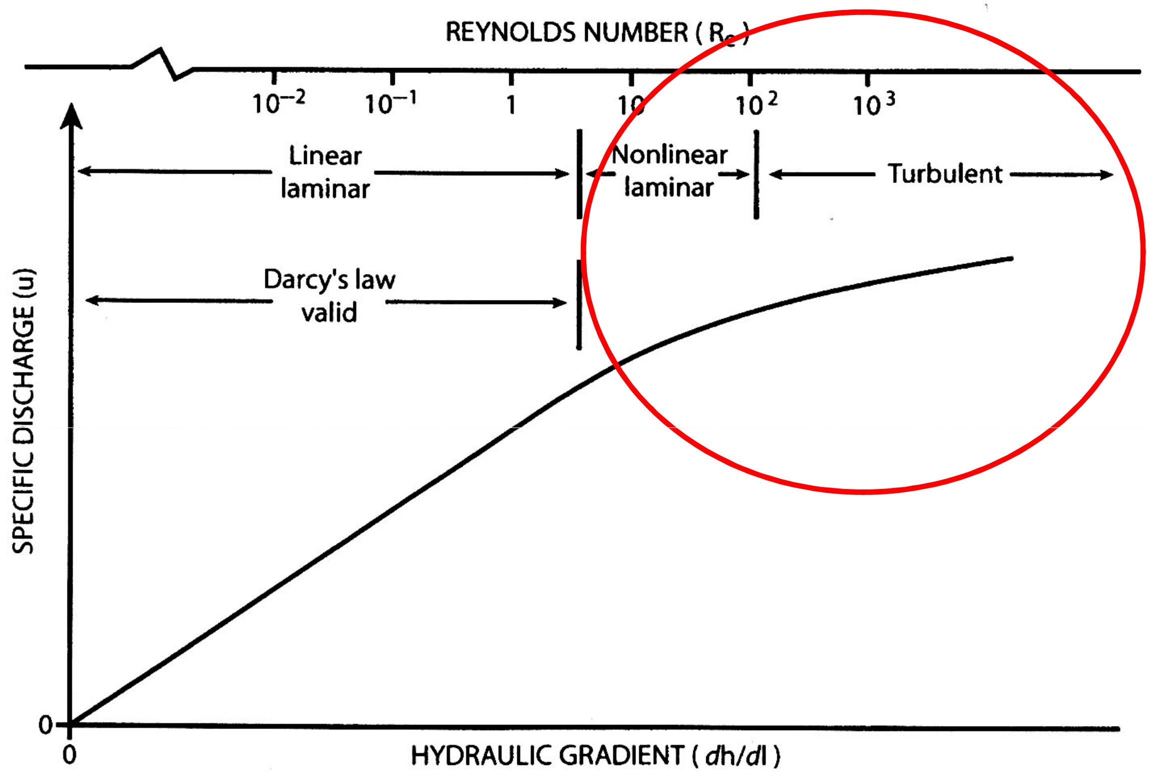


Figure 5.32 Range of validity of Darcy's Law. Reproduced from Freeze, R.A. and J.A. Cherry, Groundwater, p. 604 © Prentice Hall, Inc.

- Darcy's Law is applicable only for laminar flow with a Reynolds number range of 1-10
- As flow exceeds R>1 - 10 the relationship between gradient and specific discharge becomes non-linear.
- Under these conditions, Darcy's Law no longer applies

The Problem with Darcy and Karst Aquifers

- In karst aquifers, Darcy's Law does not apply
  - Flow is often turbulent in caves/conduits
- Models that only use Darcy's Law are prone to error when applied to karst aquifers
  - Are also less physically realistic
  - Do not account for karst parameters
    - Pipe diameter, tortuosity, etc.
- Other methods have been developed to model karst aquifers (e.g. MODFLOW-CFP)
  - Couple Darcy's law with pipe flow equations
  - Account for turbulent flow and pipe parameters
- New methods in karst modeling need to be evaluated to determine the limits of their accuracy and usefulness

Purpose

- To evaluate the new groundwater modeling program MODFLOW-CFP against MODFLOW-2005, to determine which program produces more accurate groundwater simulation results of karst aquifers.
- To develop more fundamental numerical method to evaluate the Pipe Flow model for conduits

An evaluation of MODFLOW-CFP at the laboratory and sub-regional scale

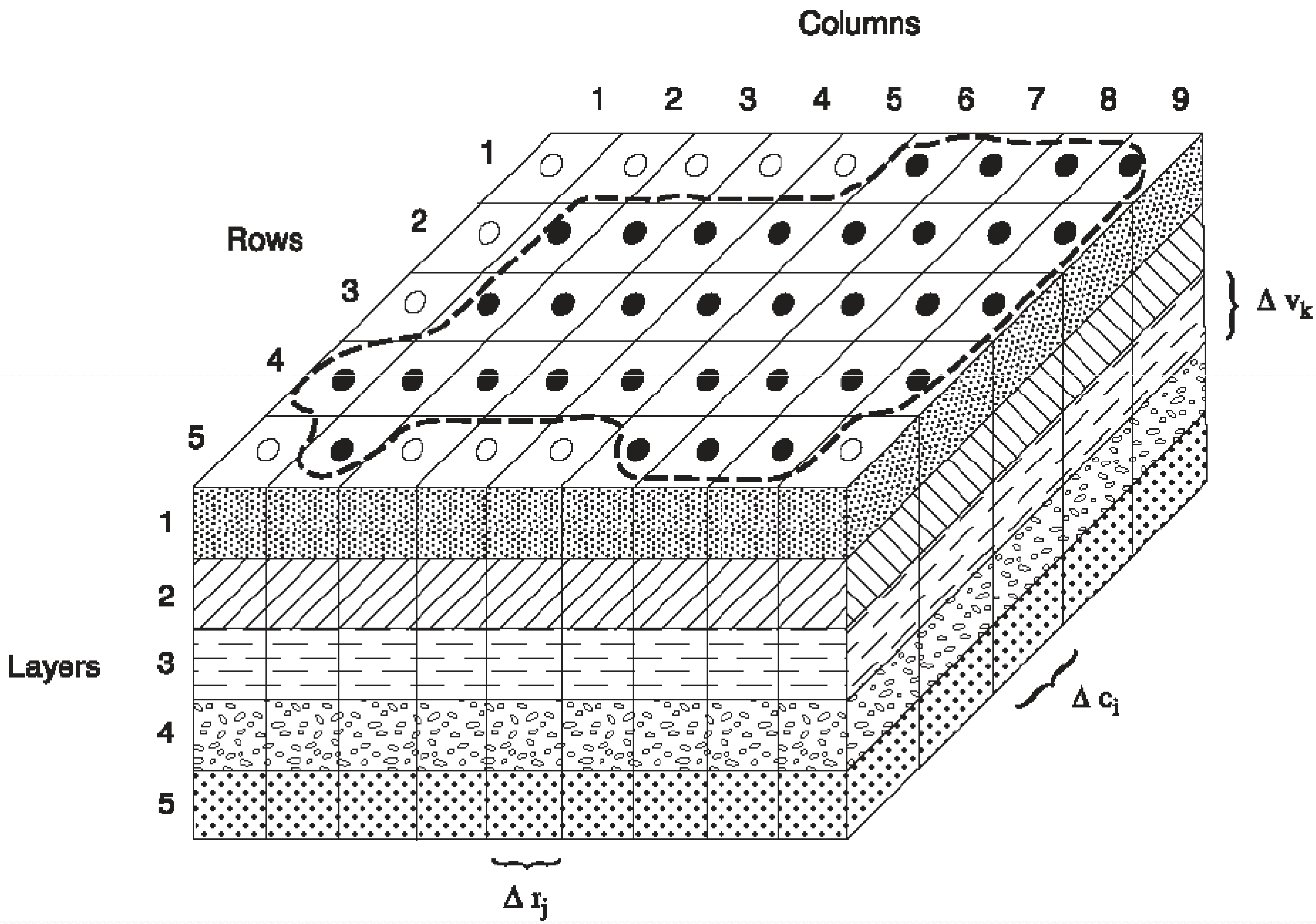
Two Groundwater Model Methods

- These two programs represent two different methods for creating groundwater models:
  - MODFLOW-2005
    - Equivalent Porous Medium Method
    - Darcy's Law applied to both the matrix and caves/conduits
    - Does not account for turbulent flow
  - MODFLOW - CFP
    - Hybrid Method
    - Darcy's Law applied to the matrix
    - Pipe equations applied to karst caves/conduits
    - Accounts for turbulent flow
    - Physically more realistic

Hypothesis

- MODFLOW-CFP is a more physically realistic groundwater modeling program, therefore it will produce more accurate results.

Methods

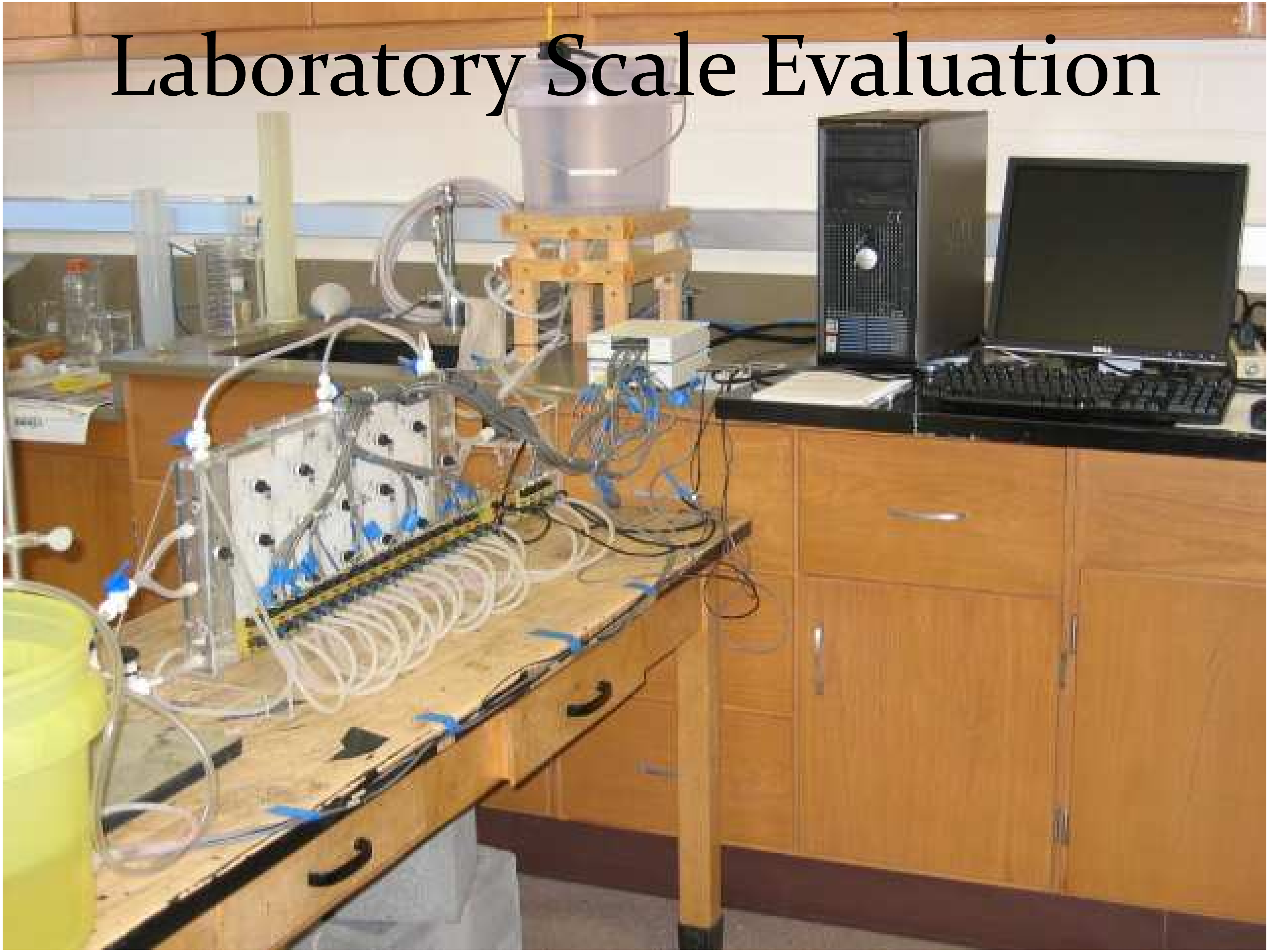


From Harbaugh, 2005

MODFLOW-CFP

- CFP is a program developed by USGS that simulates flow in a dual porosity aquifer
- Meant to work in conjunction with MODFLOW-2005
- There are 3 modes
  - CFPM1
    - simulates flow in karst aquifers with caves/conduits e.g. Wakulla Sp.
  - CFPM2
    - simulates flow in karst aquifers with preferential flow layers caused by vuggy porosity e.g. Biscayne aquifer
  - CFPM3 – combination of the two modes above

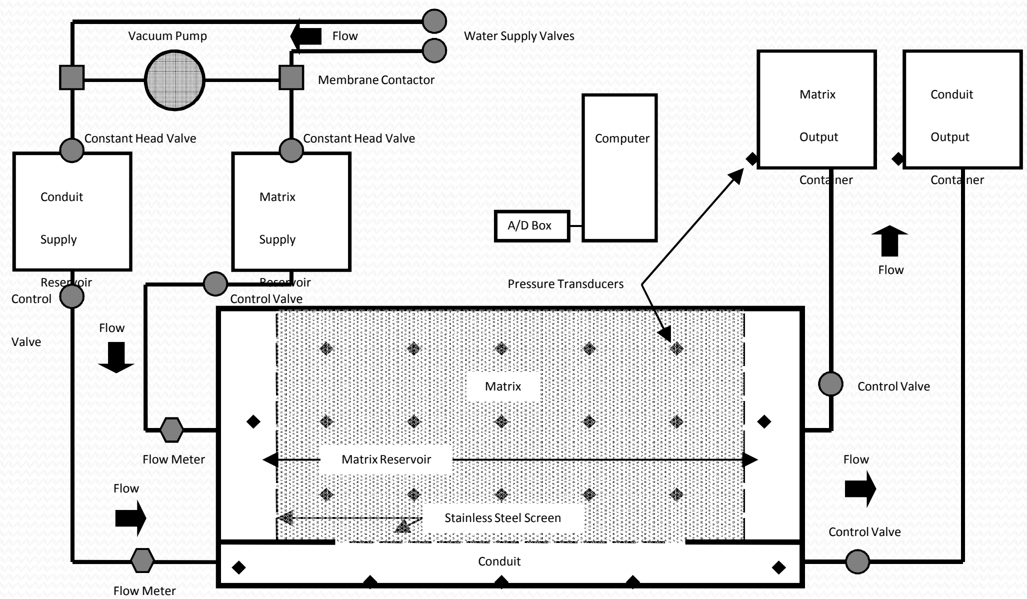
Laboratory Scale Evaluation



Karst Analog Model

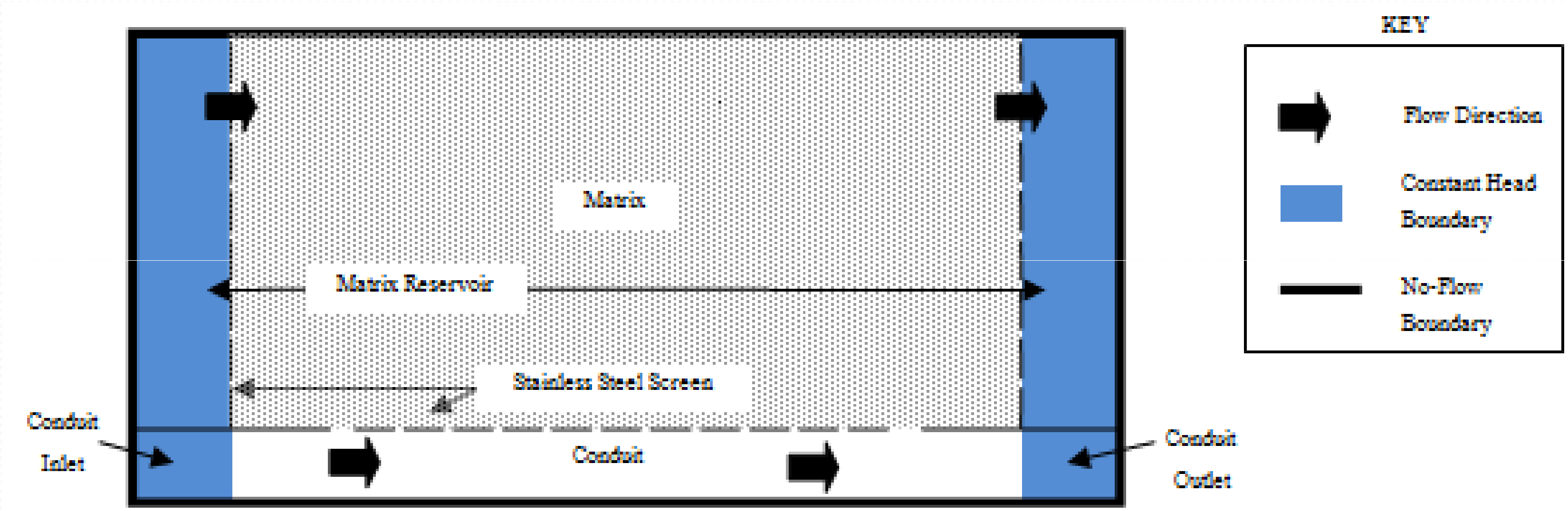
- Faulkner et al. (2009) created a laboratory scale analog model of a karst aquifer that simulated flow & transport within the conduit and matrix domains

- Two general cases were examined:
  - Conduit Head>Matrix Head
  - Matrix Head > Conduit Head



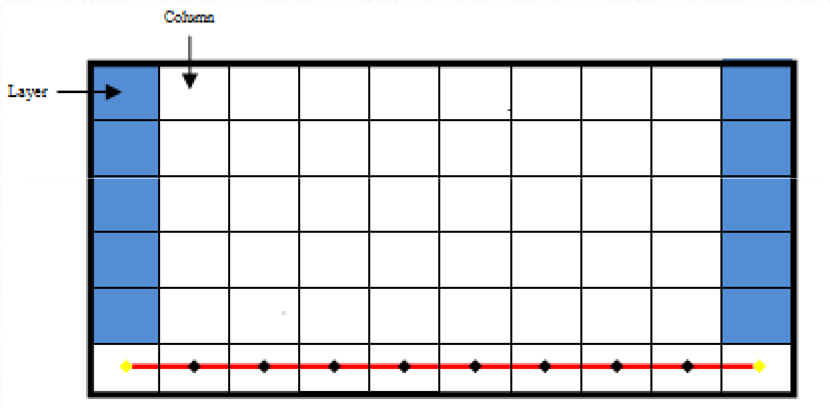
From Faulkner et al., 2009

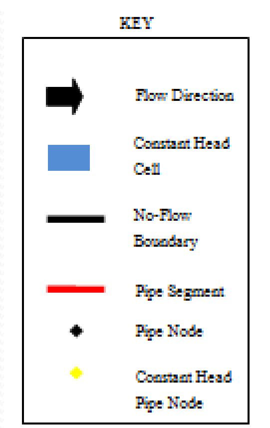
Conceptual Model



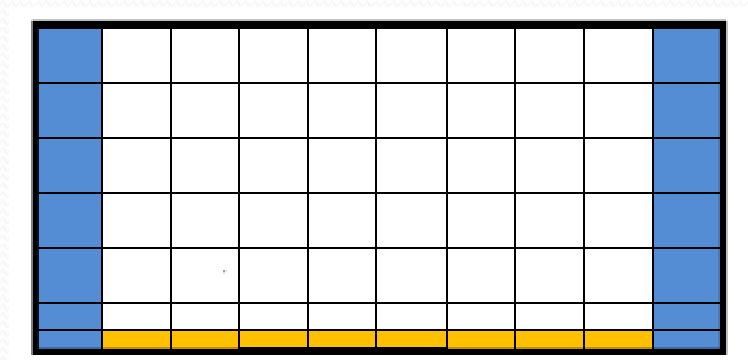
Numerical Model

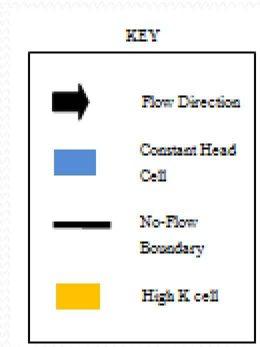
MODFLOW – CFP





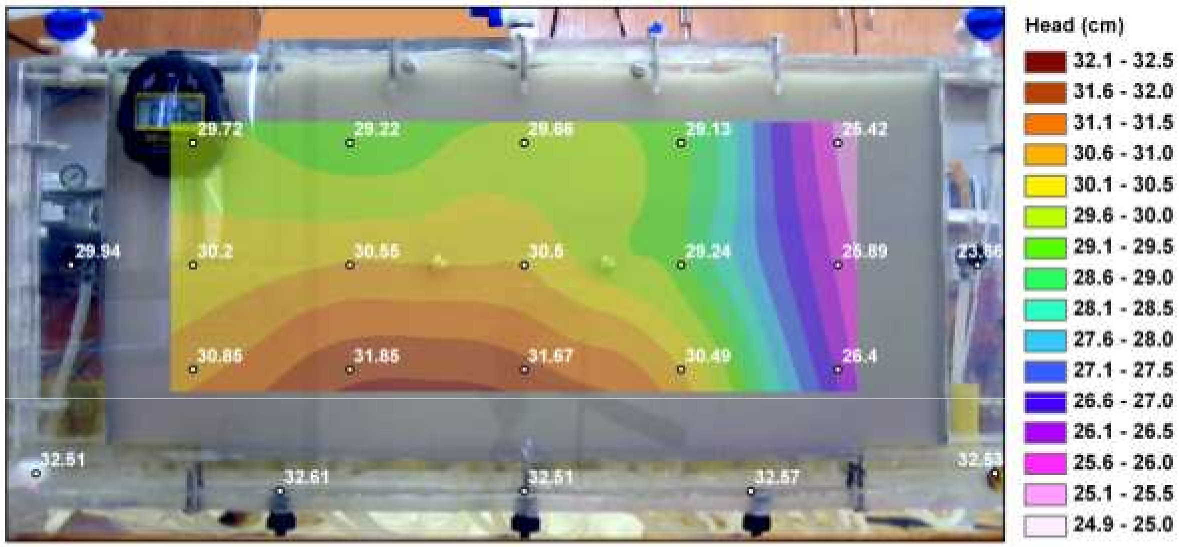
MODFLOW – 2005



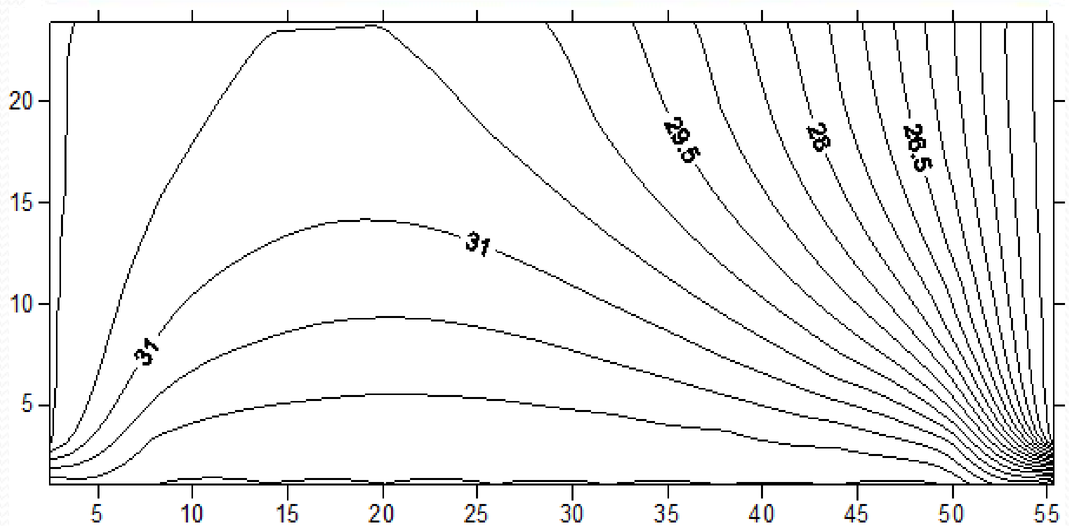


MF2K5: Conduit Head > Matrix Head

Analog Data

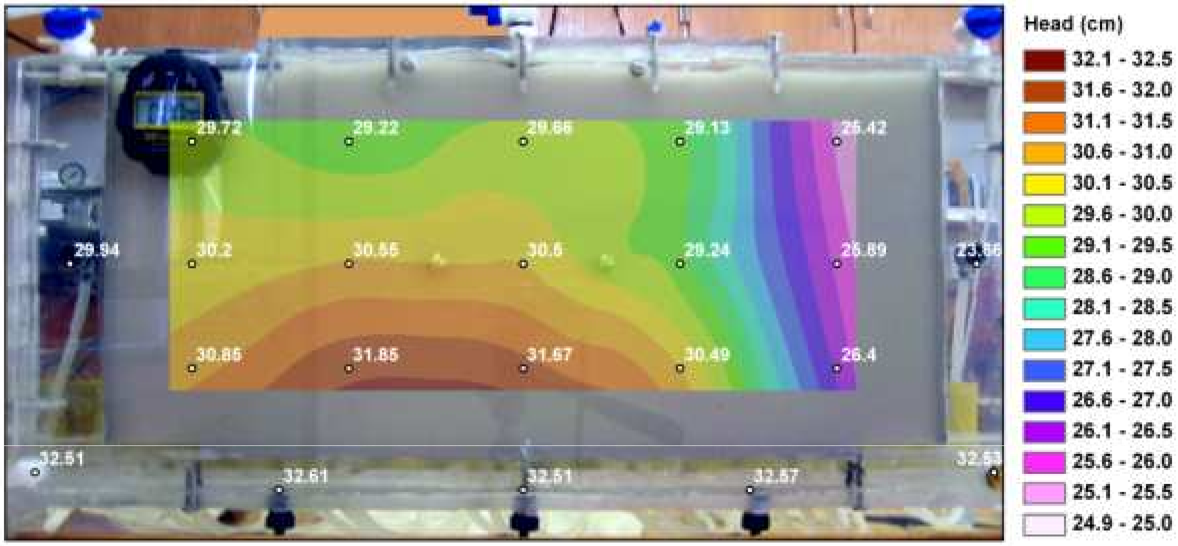


Modelflow 2005 Simulation

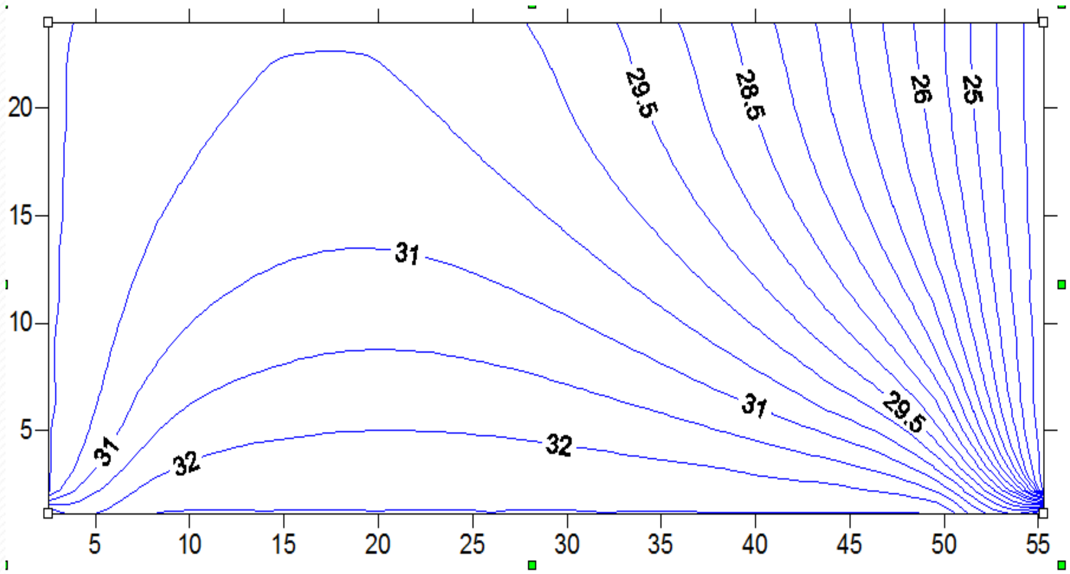


CFP : Conduit Head > Matrix Head

Analog Data



CFP Simulation

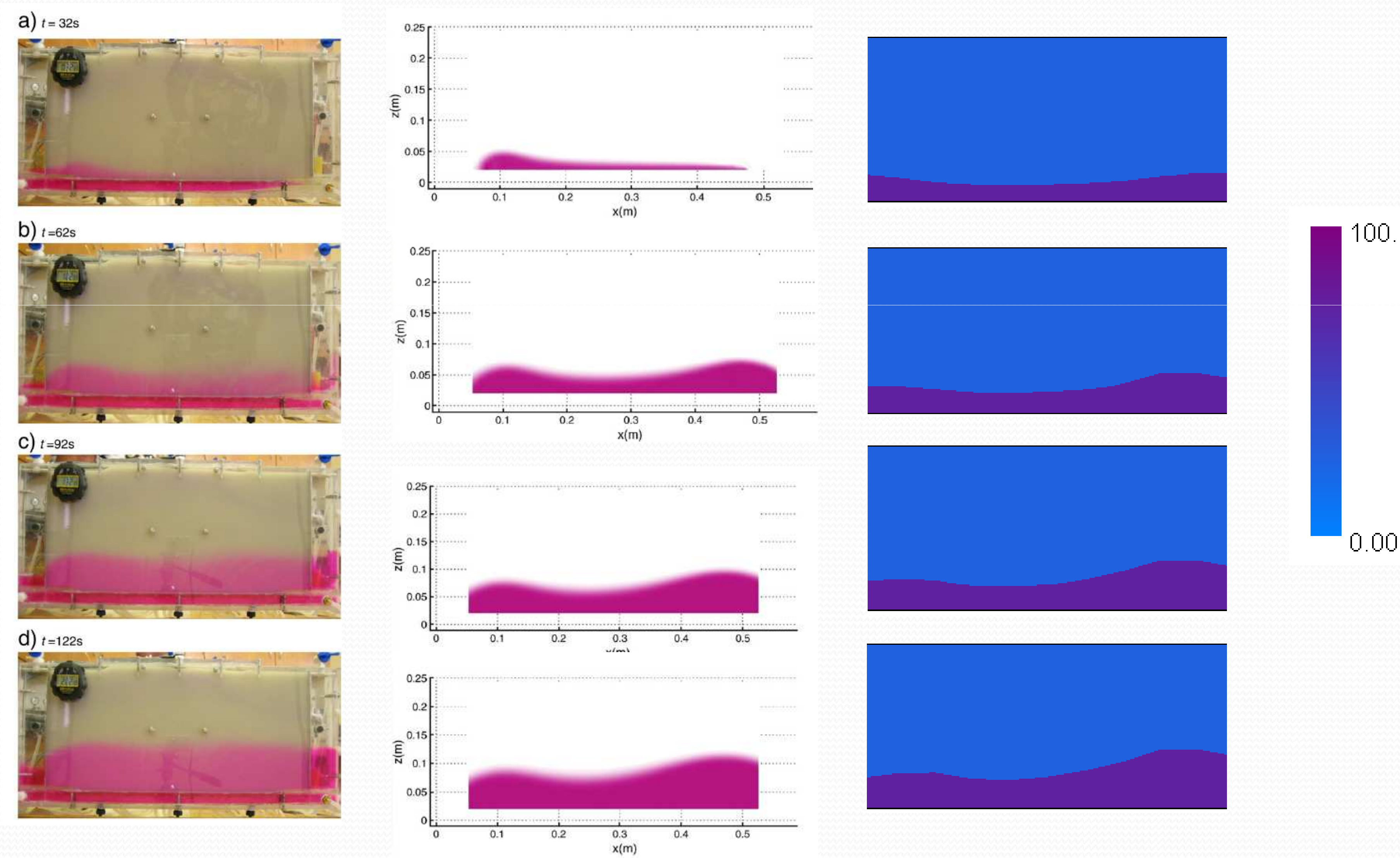


Residual Comparison

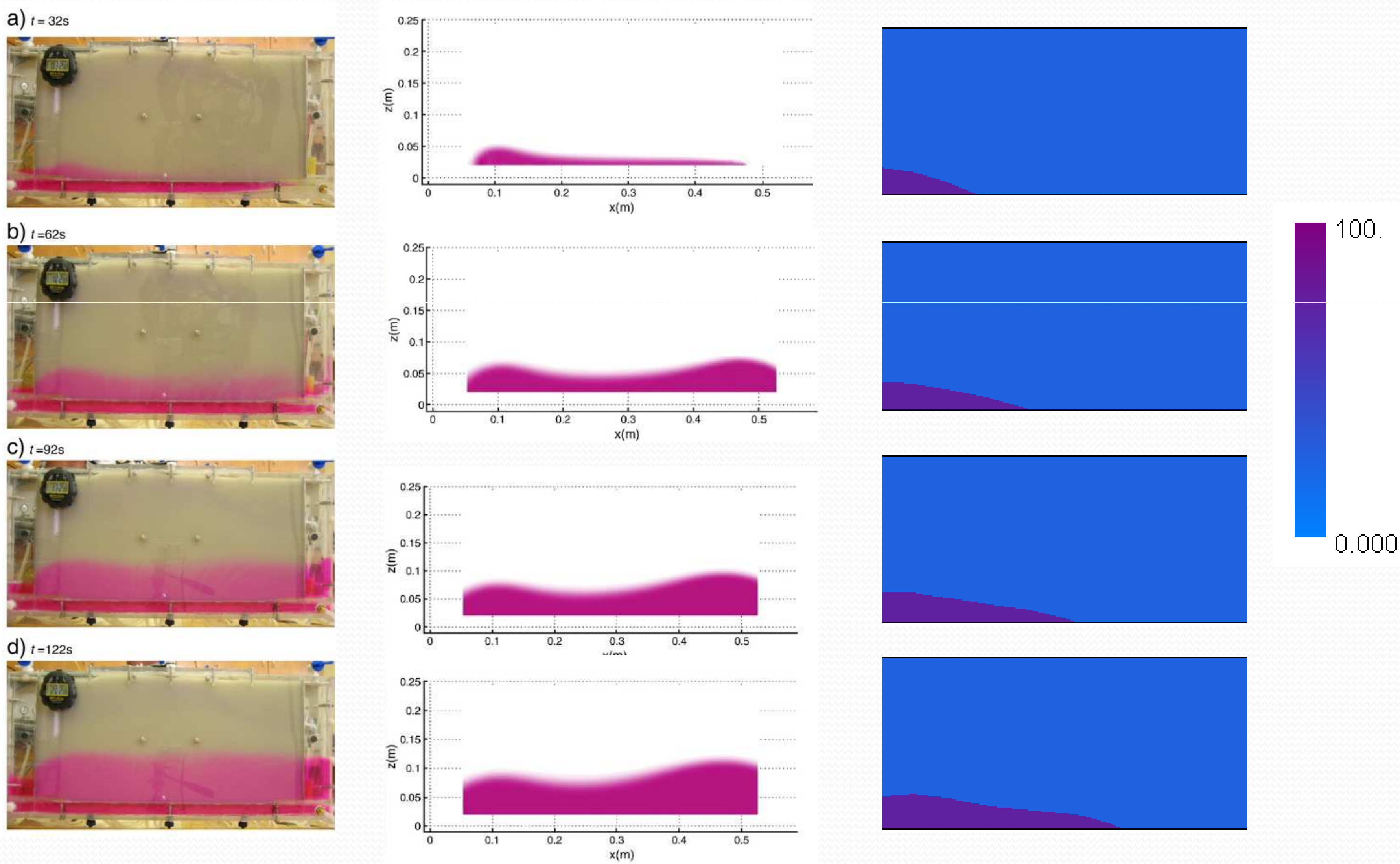
- For comparison, residuals for each flow model were calculated
- CFP exhibited the best performance

Model Type	Residual Mean	Residual Absolute Mean	Residual Variance
CFP	-0.55	0.63	0.59
Modflow 2005	-0.61	0.66	0.65

Analog vs. CFP/MT3DMS



Analog vs. Modflow/ MT3DMS

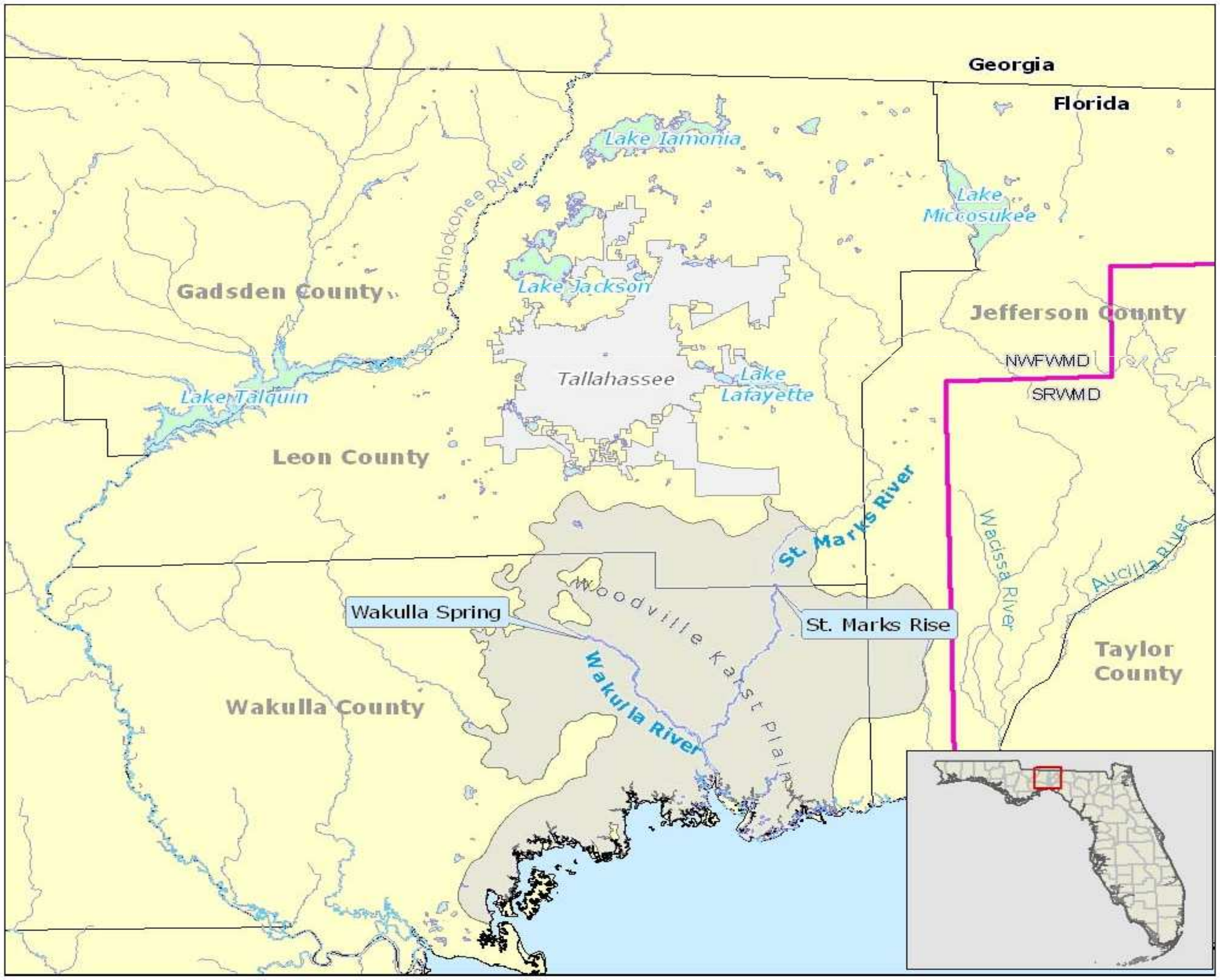


Conclusions

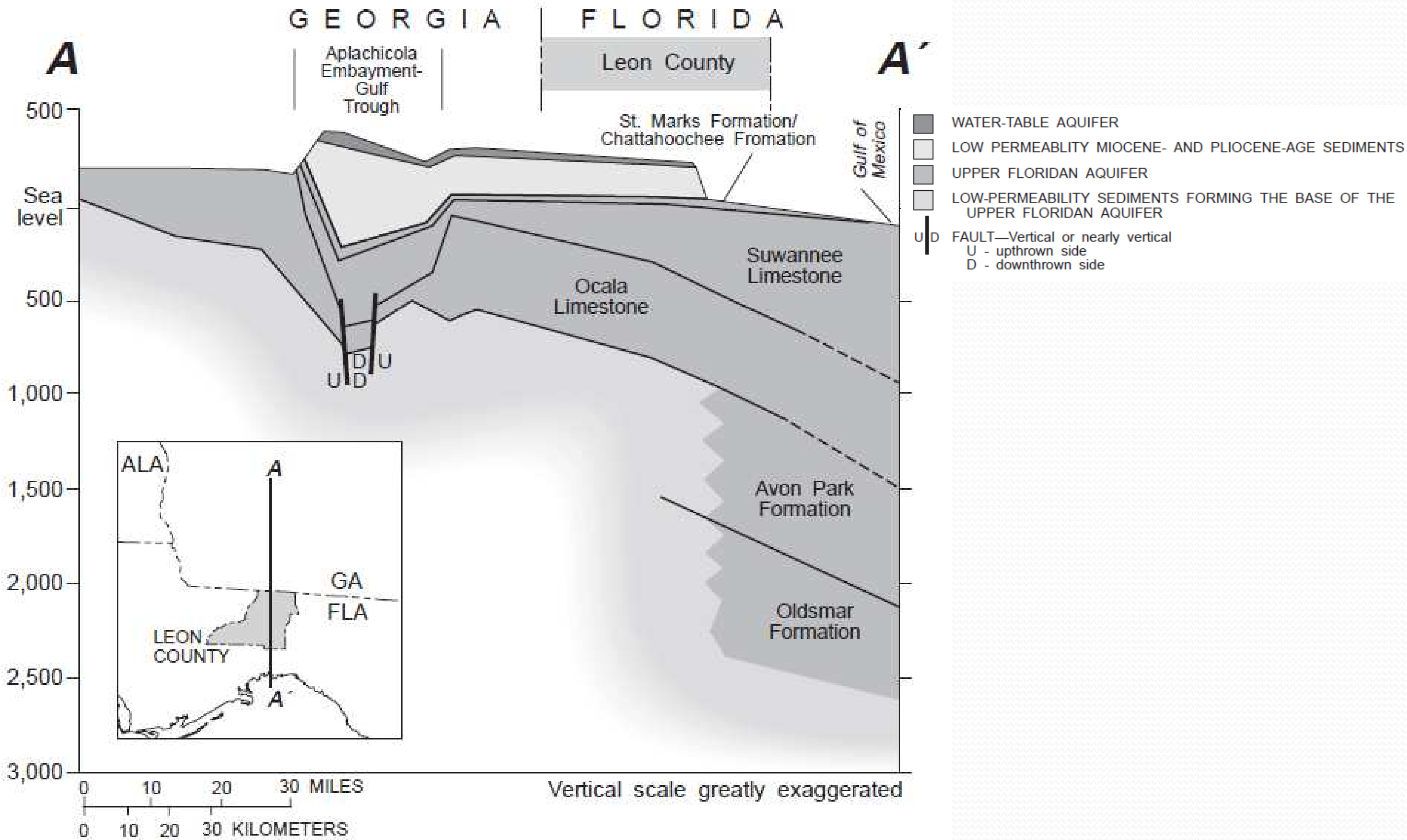
- Laboratory Scale
  - CFP flow model performs better than MODFLOW-2005
  - CFP and MT3DMS can simulate transport in a simple conduit
  - MODFLOW coupled with MT3DMS simulates transport poorly

Sub-Regional Scale Evaluation

Woodville Karst Plain, Florida

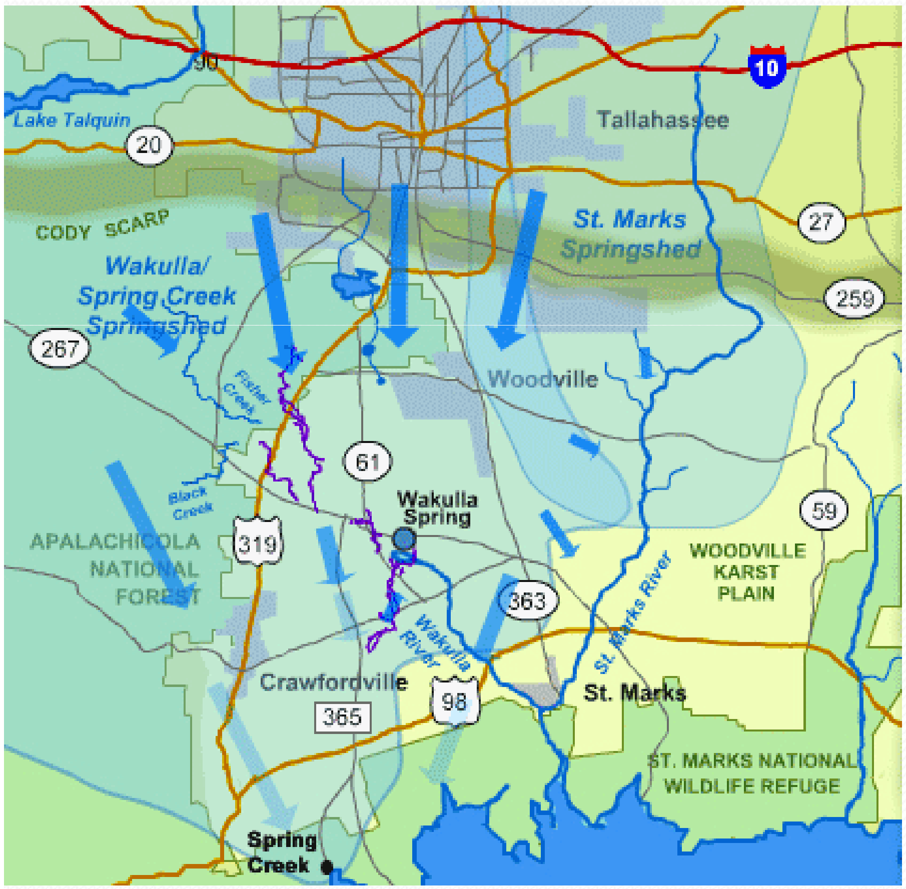


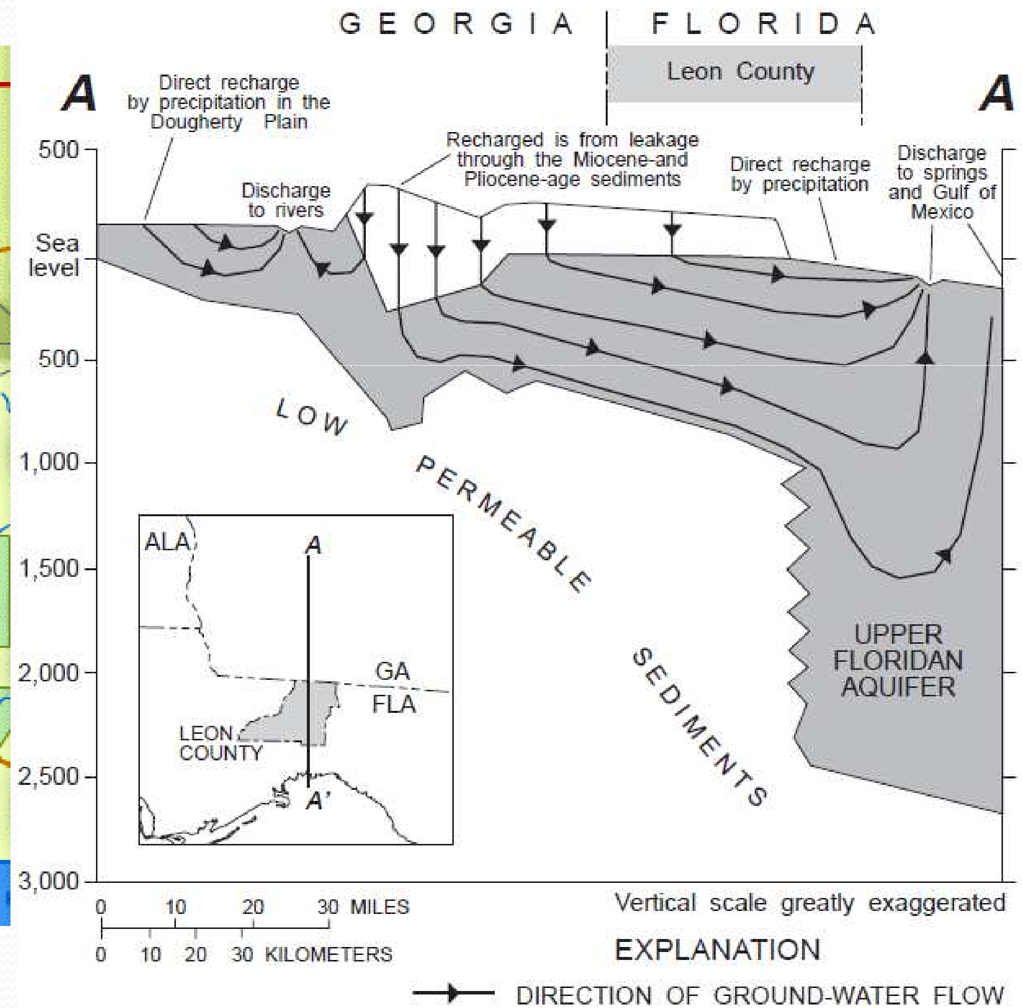
Woodville Karst Plain Geology



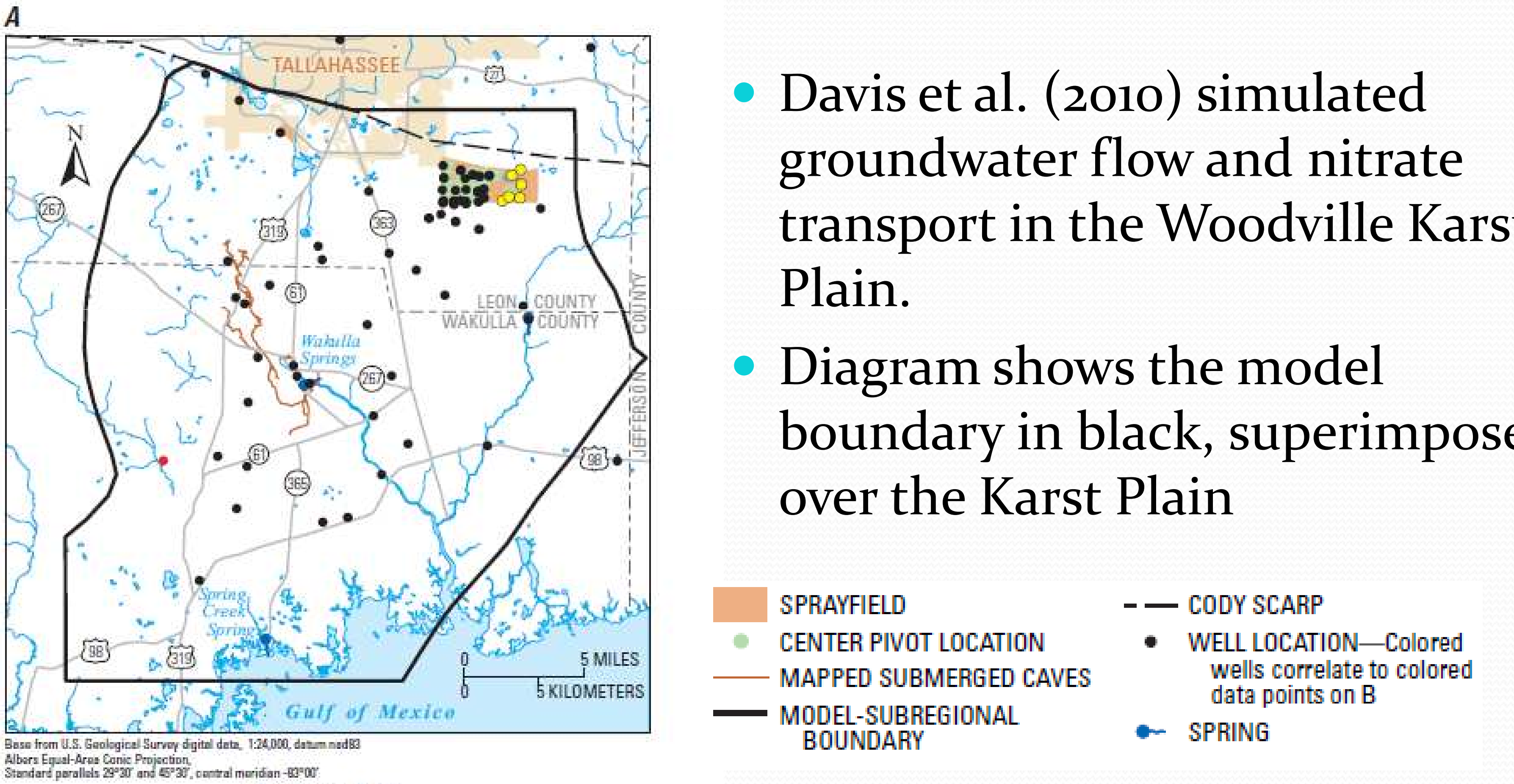
Karst Features

Groundwater Flow



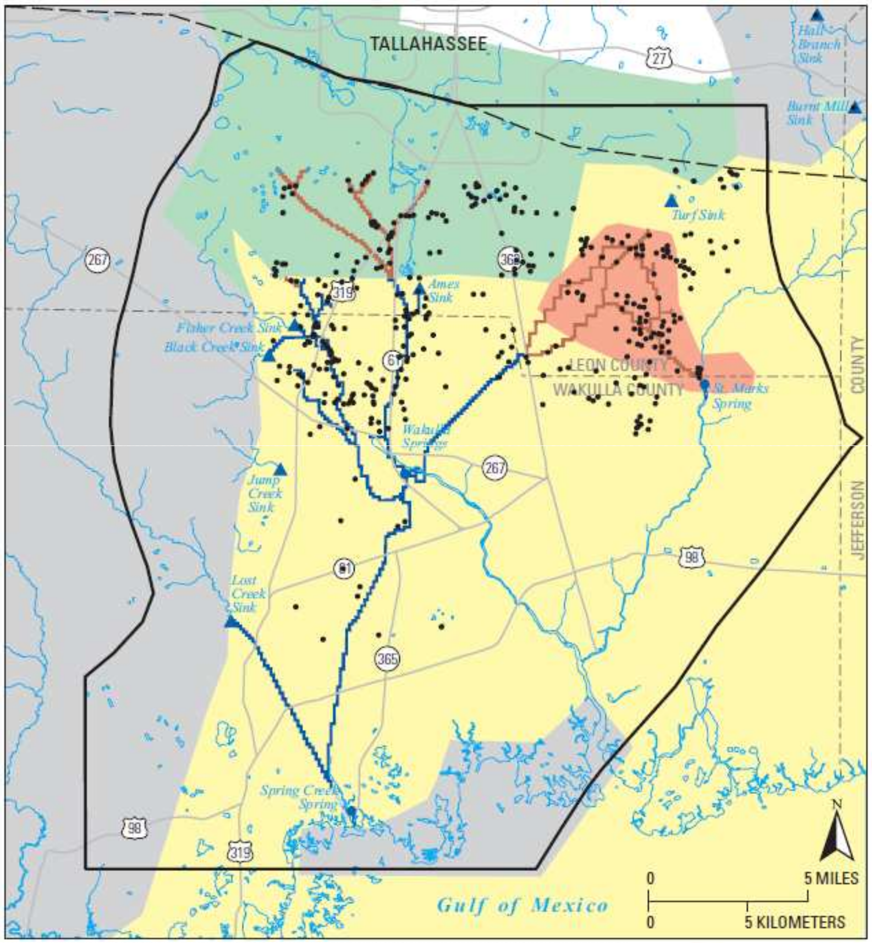


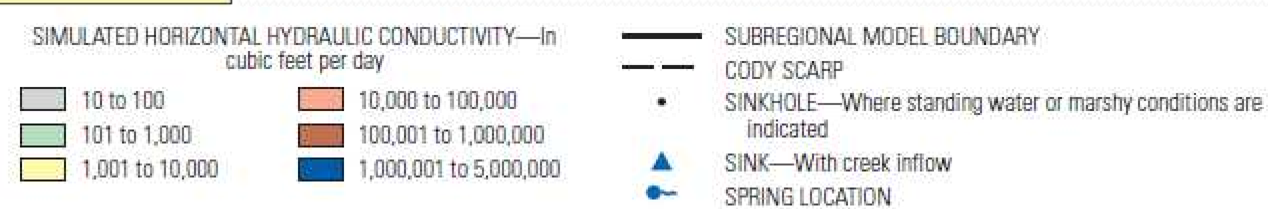
Woodville Karst Plain Model



- Davis et al. (2010) simulated groundwater flow and nitrate transport in the Woodville Karst Plain.
- Diagram shows the model boundary in black, superimposed over the Karst Plain

Simulation of Flow using MODFLOW

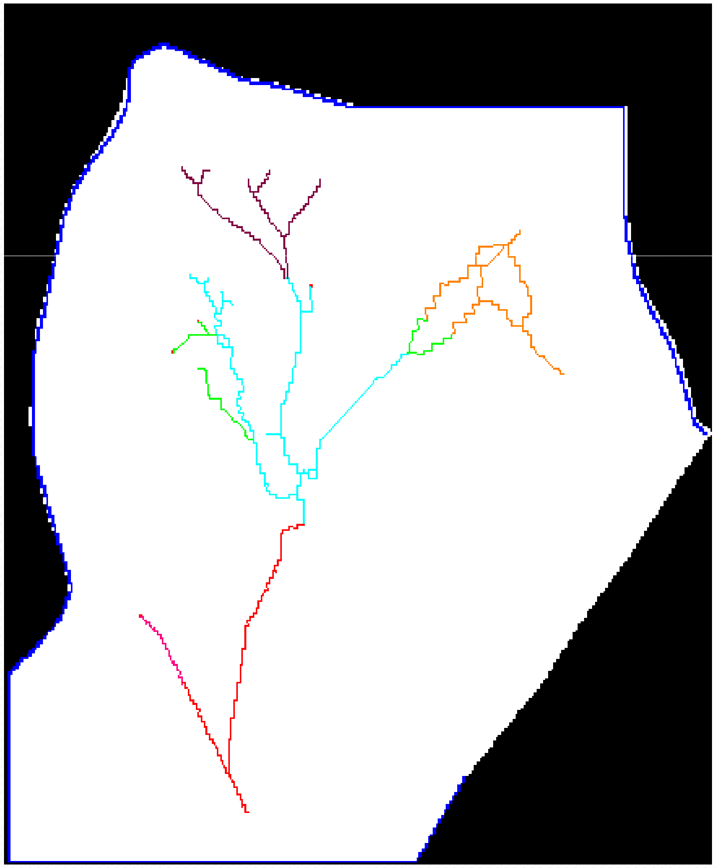




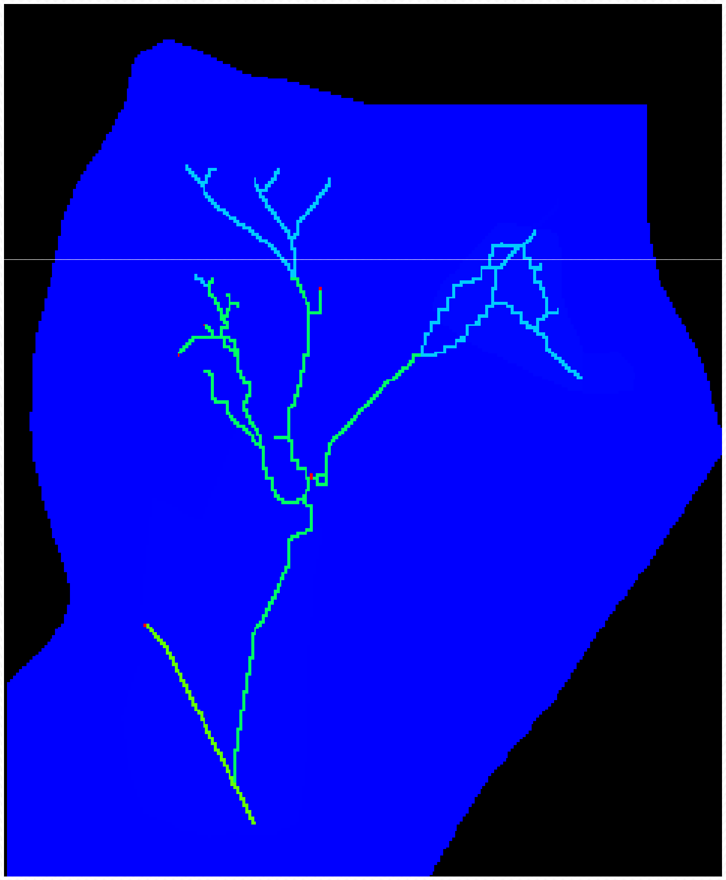
- Flow through caves and conduits was simulated by assigning high K values to cells that contained caves.
- Model was calibrated to discharge of springs by varying K of cells surrounding caves.

Simulation of Flow using CFP

- A CFP version was created based on Davis' model but with the addition of conduits in place of the high K cells

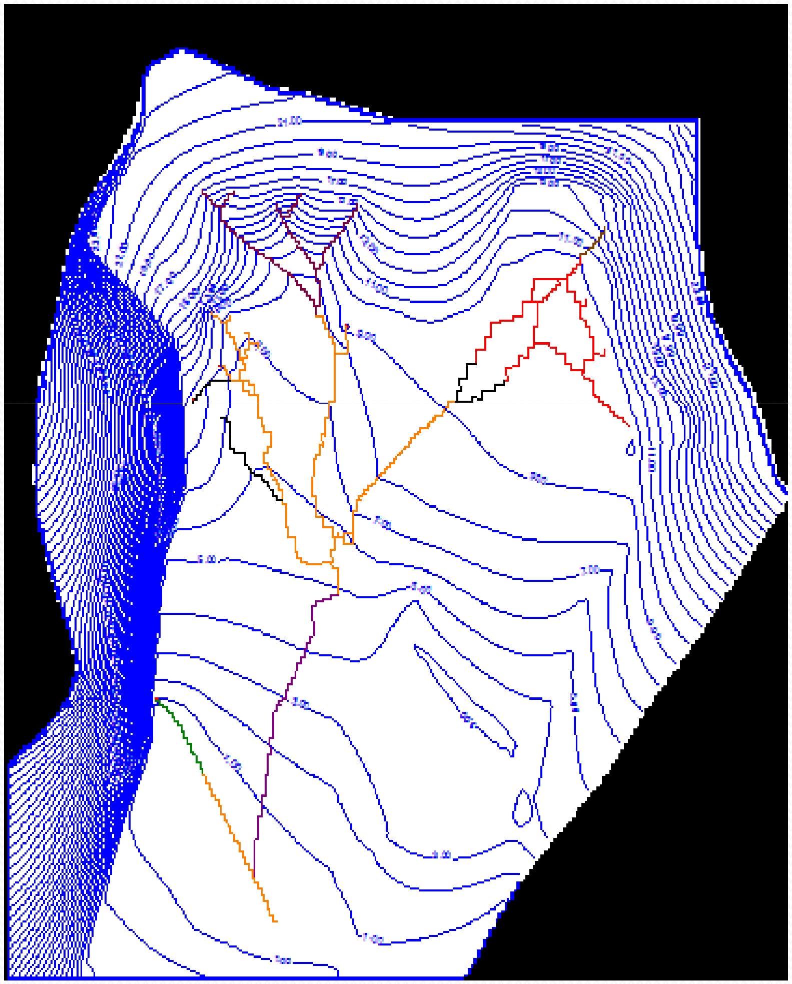


CFP model

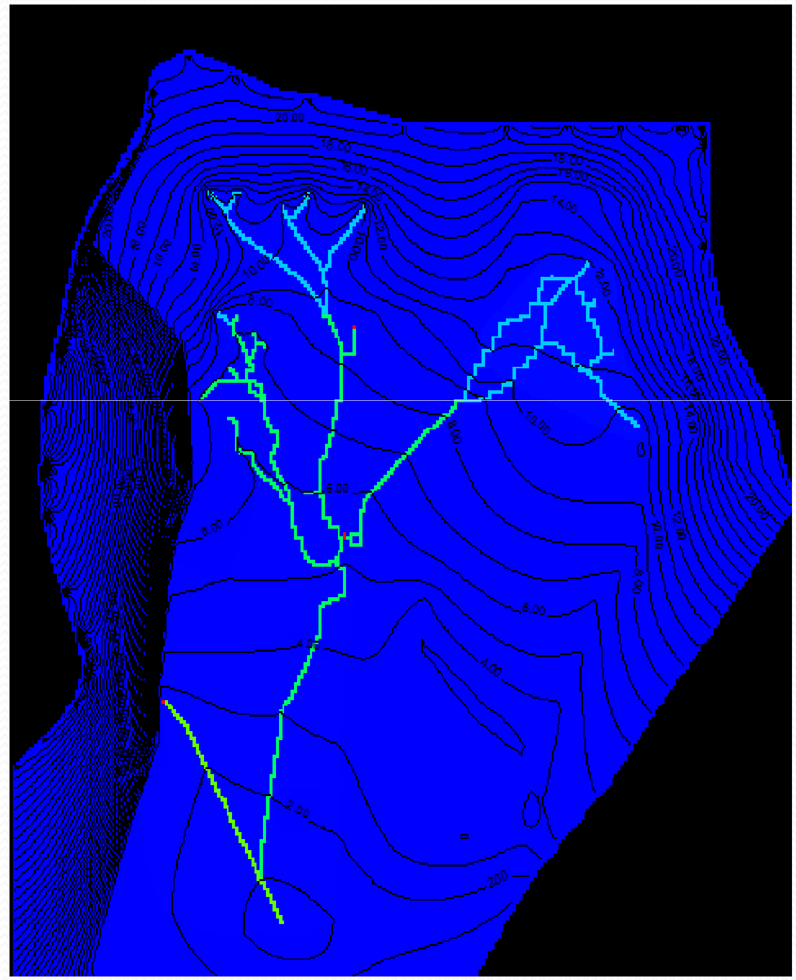


MODFLOW 2005 model

Steady State Comparison

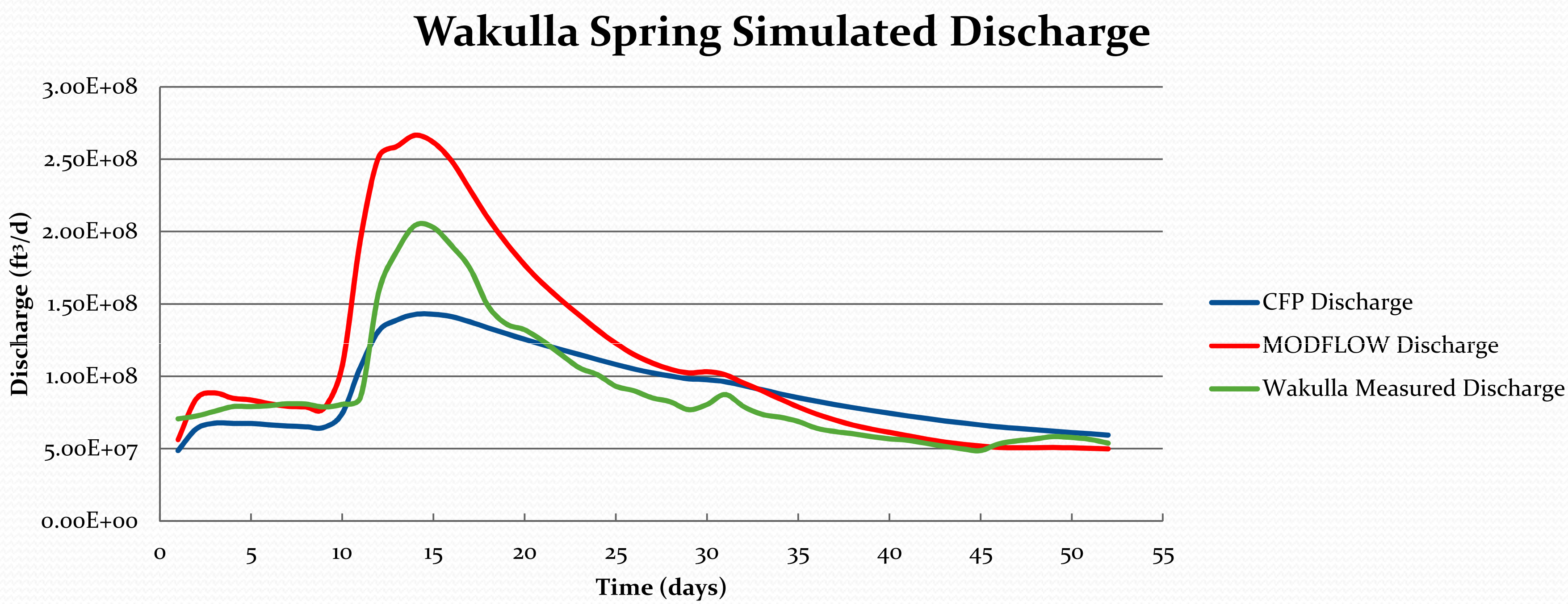


CFP model



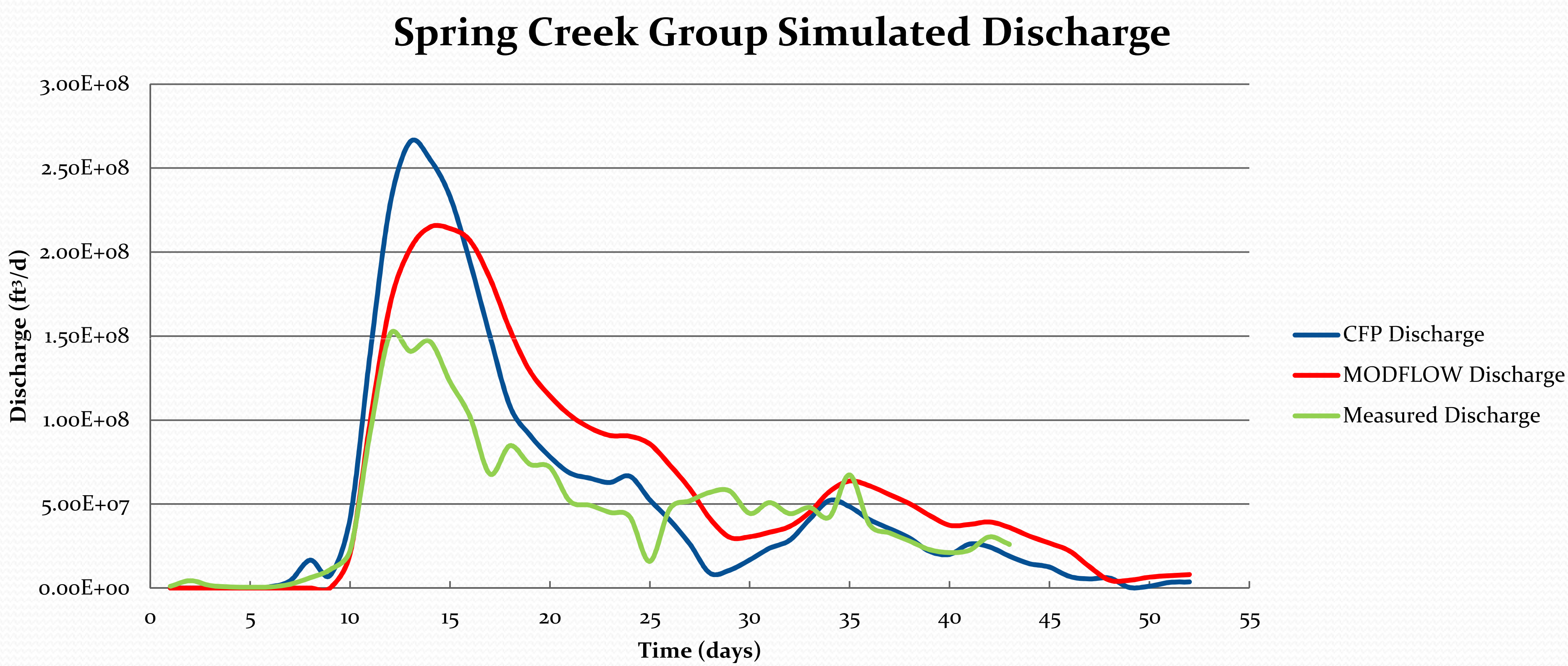
MODFLOW 2005 model

Transient State Comparison



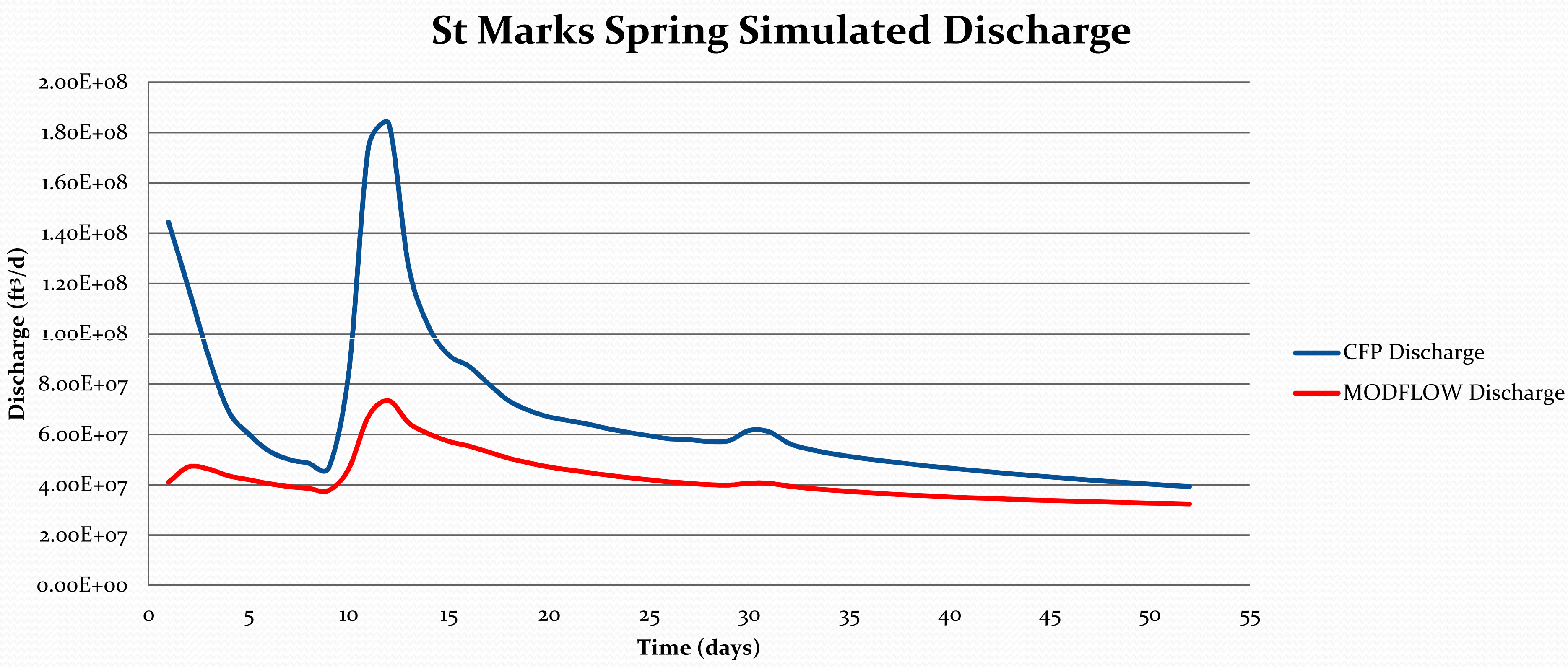
Model	% Error at Peak Discharge	Total Absolute Residual (cfd)	% Difference
CFP	-30.1%	8.8E+8	-30.91 %
MODFLOW	30.5%	1.2E+9	

Transient State Comparison



Model	% Error at Peak Discharge	Total Absolute Residual (cfd)	% Difference
CFP	75.8%	2.28E+14	-12.69 %
MODFLOW	42.2%	2.59E+14	

Transient State Comparison



Conclusions

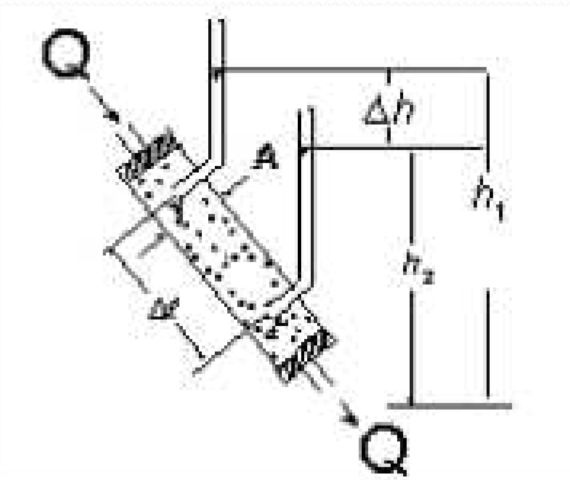
- Sub-Regional Scale
  - CFP can simulate flow in karst aquifers at a sub-regional scale under steady state conditions
  - Under Transient conditions, MODFLOW-2005 more accurately simulates peak discharge
  - Overall however, total residuals show that MODFLOW-CFP is producing more accurate results

Development of Stokes-Darcy Model for Groundwater Flow and Solute Transport in a Karst Aquifer

The flow in the matrix Ω_m is governed by the Darcy's law:

$$S\frac{\partial \phi_m}{\partial t} + \nabla \cdot \mathbf{v}_m = 0, \qquad (1)$$

$$\mathbf{v}_m = -\boldsymbol{K}\nabla \phi_m, \qquad (2)$$



ϕ_m hydraulic (piezometric) head

defined as $z + \frac{p_m}{\rho g}$

$\mathbf{v}_m$ discharge rate (Q)

S mass storage coefficient

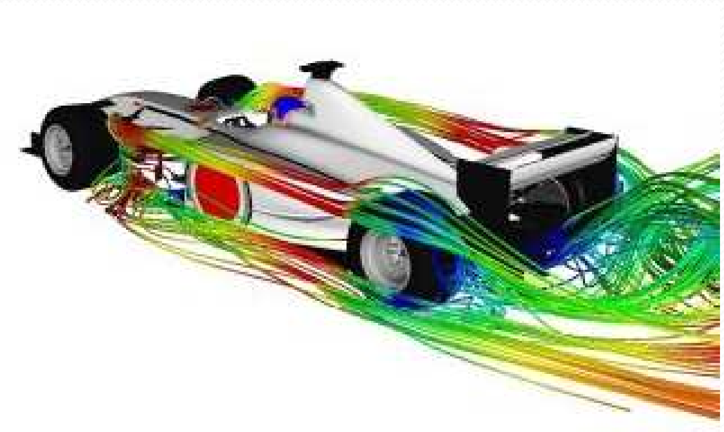
$\boldsymbol{K}$ hydraulic conductivity tensor (SPD)

p_m dynamic pressure

In the conduit, denoted by Ω_C , the Navier-Stokes equations govern the free flow:

$$\overbrace{\underbrace{\frac{\partial \mathbf{v}_c}{\partial t}}_{\substack{unsteady\\acceleration}} + \underbrace{\mathbf{v}_c \cdot \nabla \mathbf{v}_c}_{\substack{convective\\acceleration}}}^{\substack{total\\derivative}} = \nabla \cdot \overbrace{(\underbrace{-p\mathbf{I}}_{pressure} + \underbrace{2\nu \boldsymbol{D}(\mathbf{v}_c)}_{deformation})}^{\substack{stress\\tensor}} - \overbrace{g\mathbf{k}}^{\substack{other\\force}}, \quad (3)$$

$$\nabla \cdot \mathbf{v}_c = 0, \quad (4)$$



$\boldsymbol{D}(\mathbf{v})$	deformation tensor
	defined as $\frac{1}{2}(\nabla\mathbf{v}+(\nabla\mathbf{v})^T)$
ν	kinematic viscosity
p	kinematic pressure

We need the following interface conditions to couple the two domains:

$$\left.\begin{aligned} \mathbf{v}_c \cdot \mathbf{n}_{cm} &= \mathbf{v}_m \cdot \mathbf{n}_{cm} \\ -\mathbf{n}_{cm}^T \boldsymbol{T}(\mathbf{v}_c, p)\mathbf{n}_{cm} &= g(\phi_m - z) \\ -\boldsymbol{\tau}^T \boldsymbol{T}(\mathbf{v}_c - p)\mathbf{n}_{cm} &= \frac{\alpha\sqrt{2g\nu}}{\sqrt{trace(\boldsymbol{K})}}\boldsymbol{\tau}\cdot(\mathbf{v}_c - \mathbf{v}_m) \end{aligned}\right\} \quad \text{on } \Gamma_{cm} \qquad (5)$$

where $\boldsymbol{\tau} = (\tau_1, \tau_2)$ represents a local orthonormal basis of the plane tangential to Γ_{cm}. The last interface equation we employ here is the

Beavers-Joseph condition. The condition essentially claims that the tangential component of the normal stress that the free flow incurs along the interface is proportional to the jump of tangential velocity over the interface.

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Beavers-Joseph condition. The condition essentially claims that the tangential component of the normal stress that the free flow incurs along the interface is proportional to the jump of tangential velocity over the interface.

The contaminate transport in the conduit-matrix region is described the by a advection-diffusion equation:

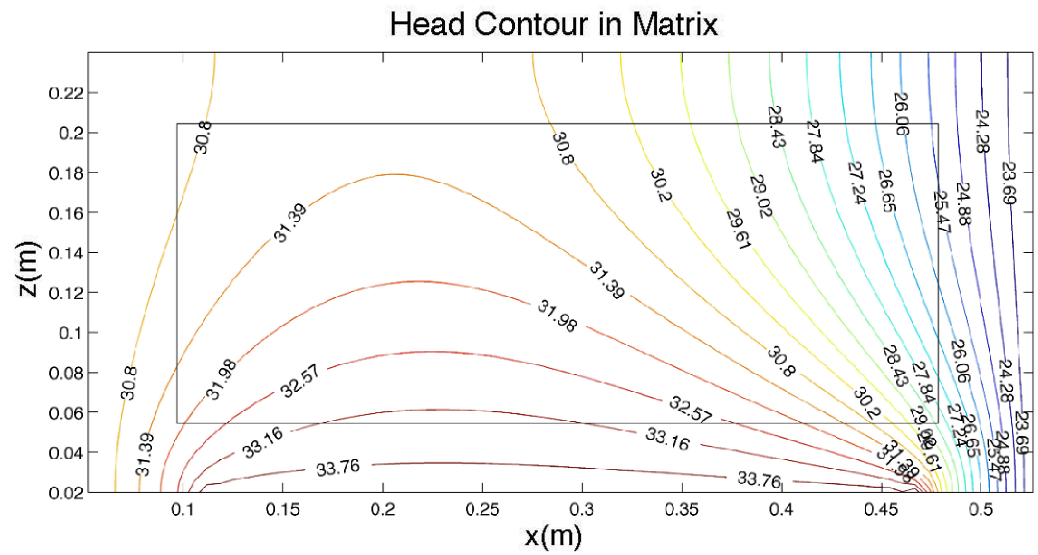
$$\frac{\partial C_m}{\partial t} + \mathbf{v}_m \cdot \nabla C_m = \nabla \cdot (\boldsymbol{D}_m \nabla C_m) \qquad in\, \Omega_m, \qquad (6)$$

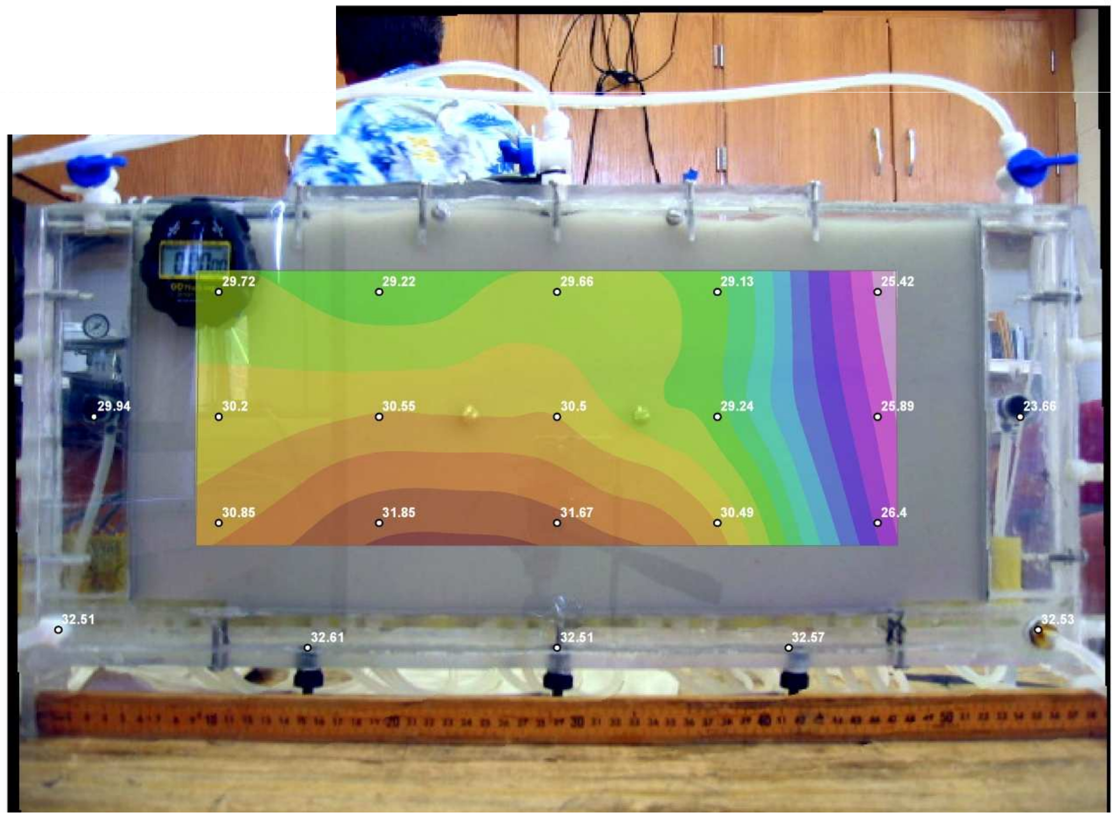
$$(\boldsymbol{D}_m \nabla C_m) \cdot \mathbf{n} = 0 \qquad on\, \Gamma_0 \cup \Gamma_g \qquad (7)$$

$$C_m = 1 \qquad flood\ season\ on\, \Gamma_{cm} \qquad (8)$$

$$(\boldsymbol{D}_m \nabla C_m) \cdot \mathbf{n}_{cm} = 0 \qquad drought\ season\ on\, \Gamma_{cm} \qquad (9)$$

where $\boldsymbol{D}_m$ denotes the diffusion tensor in the matrix. As the boundary condition for ϕ_m varies during flood season and during drought season. The water in the matrix will flow in almost opposite directions, which results in the phenomenon that the matrix serves as a storage for solute.


Head Contour in Matrix
z(m)
x(m)
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
30.8
31.39
31.98
32.57
33.16
33.76
30.2
29.61
29.02
28.43
27.84
27.24
26.65
26.06
25.47
24.88
24.28
23.69


29.72
29.22
29.66
29.13
25.42
29.94
30.2
30.55
30.5
29.24
25.89
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31.67
30.49
26.4
32.51
32.61
32.51
32.57
32.53

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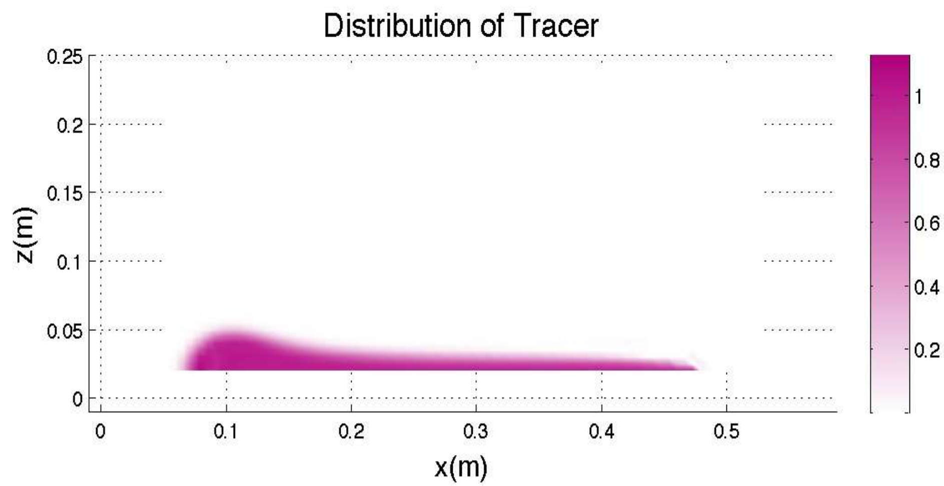
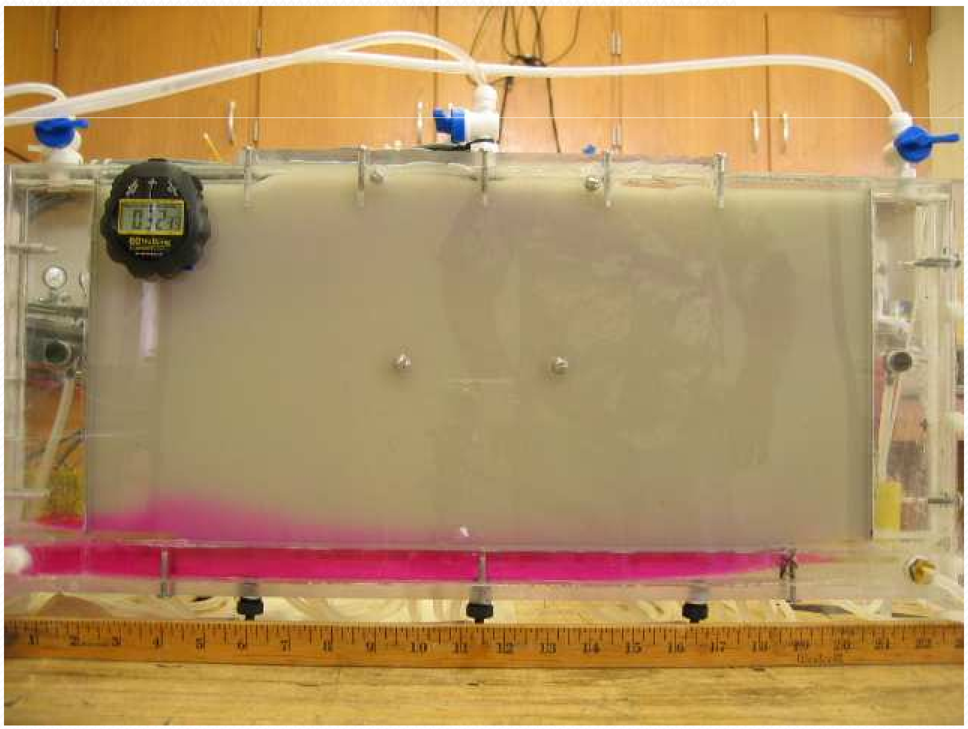


Distribution of Tracer
0.25
0.2
0.15
0.1
0.05
0
z(m)
0
0.1
0.2
0.3
0.4
0.5
x(m)
1
0.8
0.6
0.4
0.2



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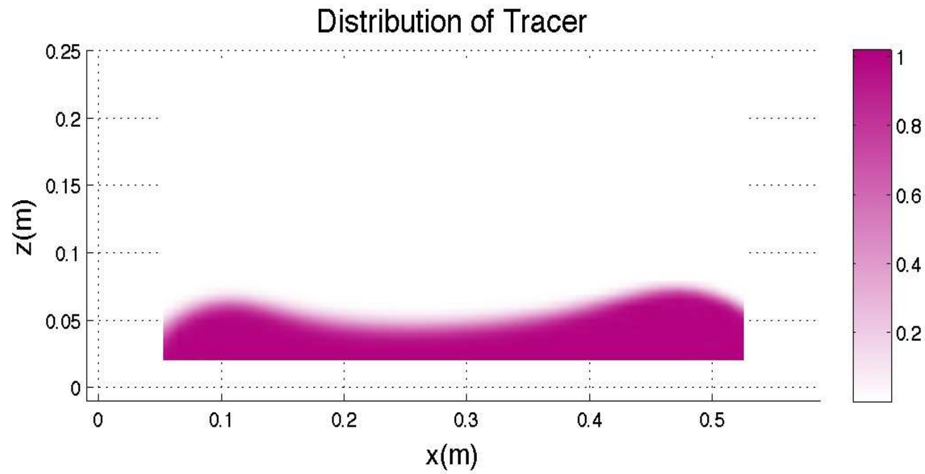
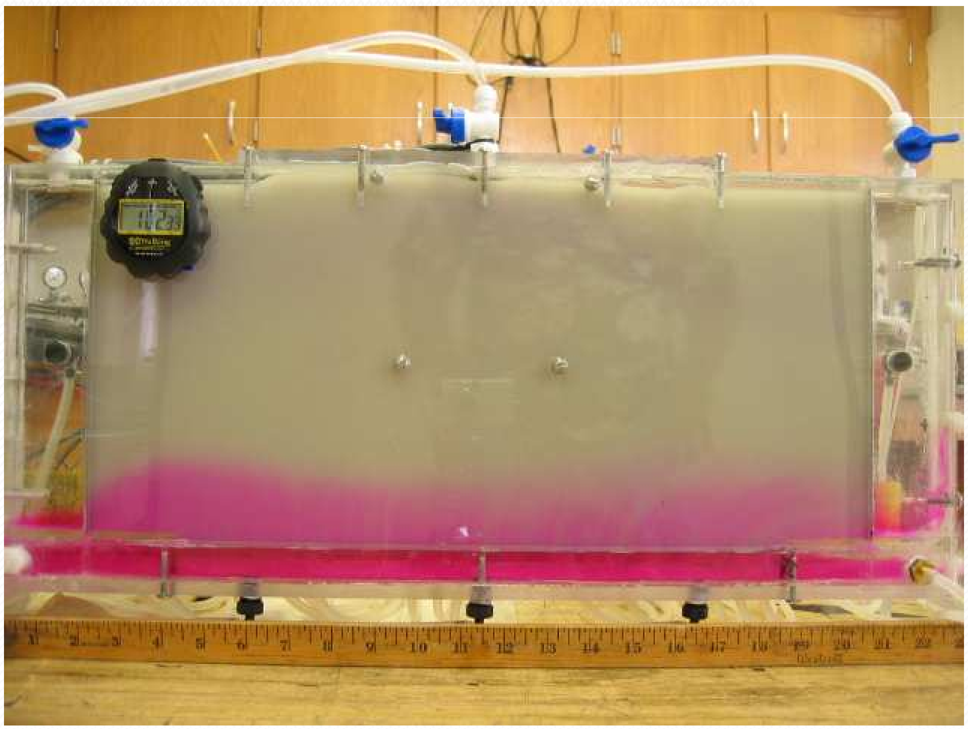

Distribution of Tracer
0.25
0.2
0.15
0.1
0.05
0
z(m)
0
0.1
0.2
0.3
0.4
0.5
x(m)
1
0.8
0.6
0.4
0.2



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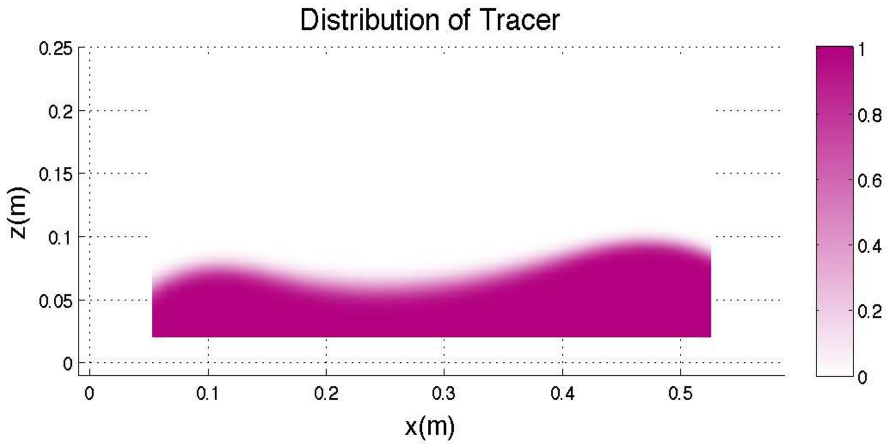
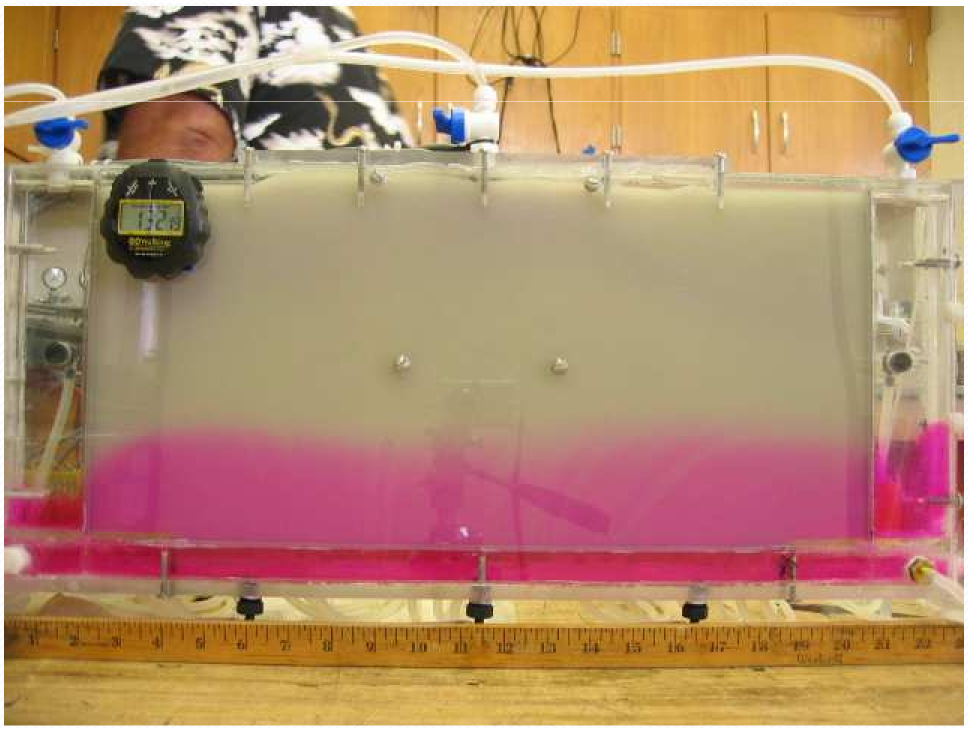
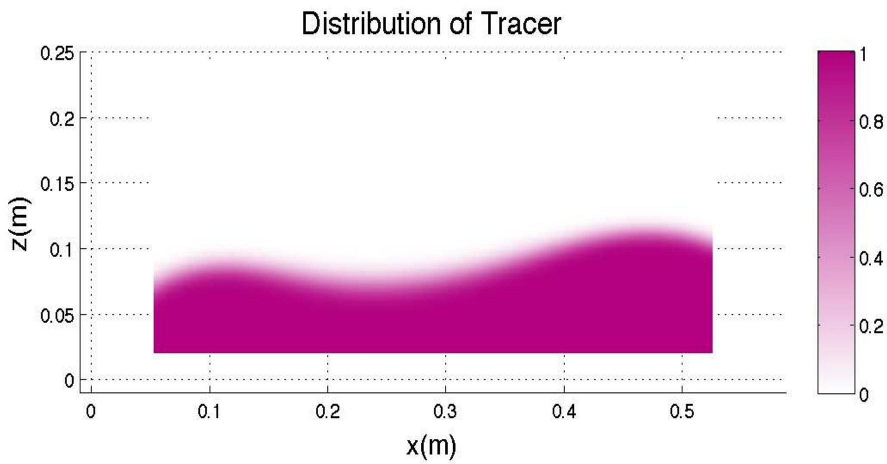
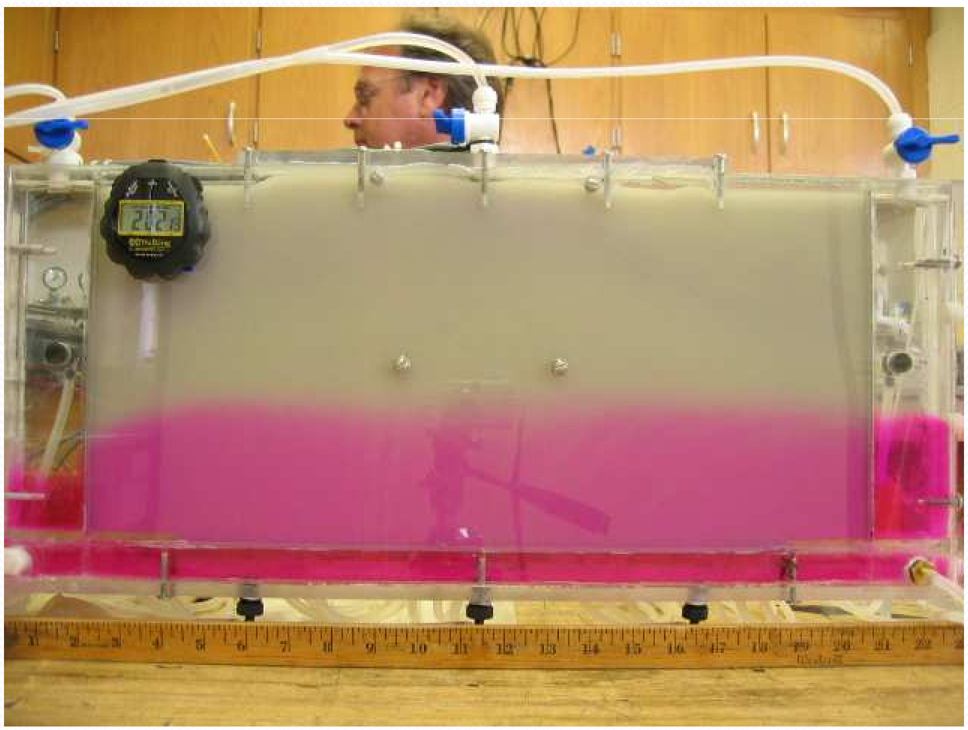

Distribution of Tracer
z(m)
x(m)
0.25
0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
1
0.8
0.6
0.4
0.2
0

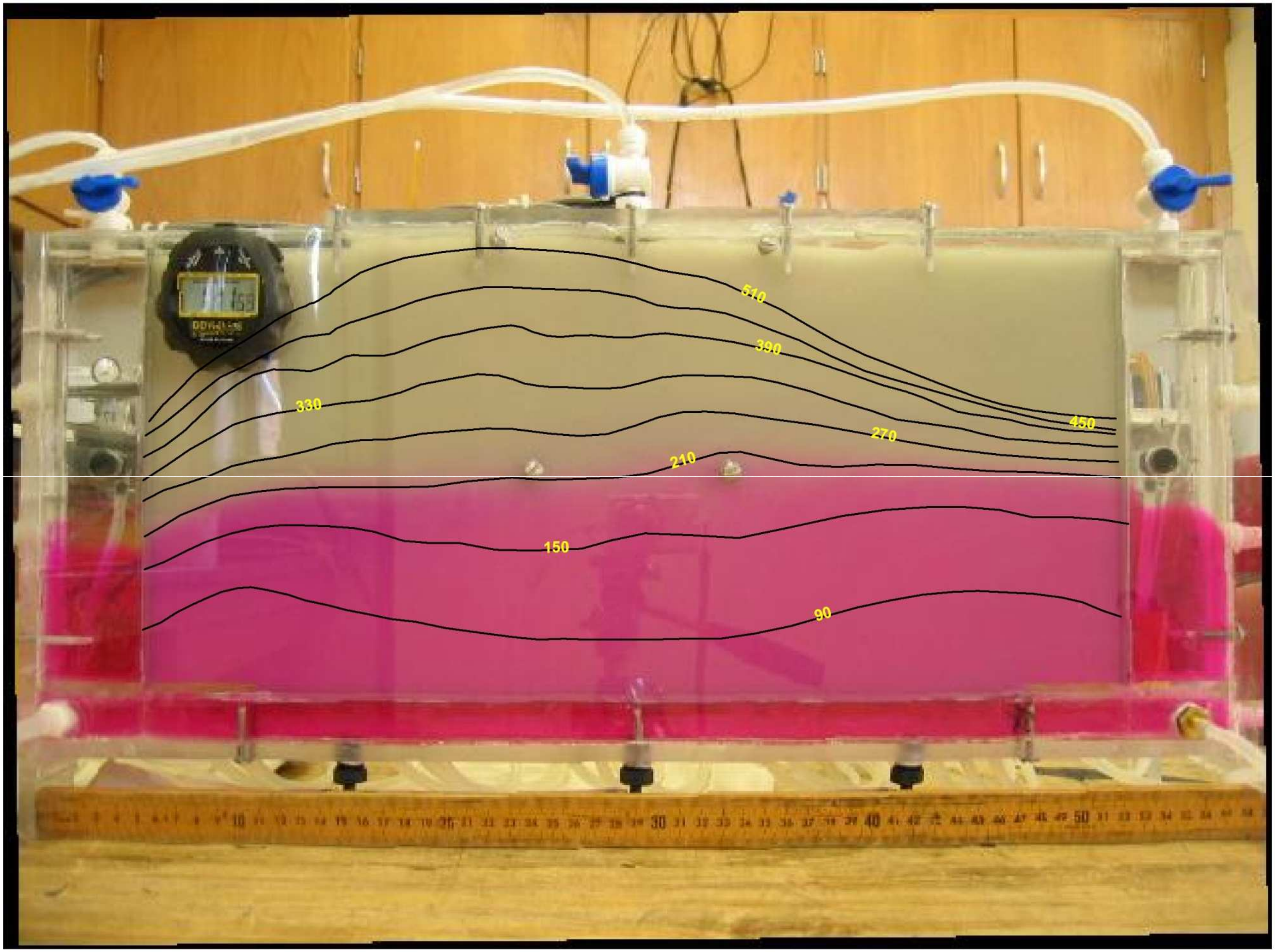


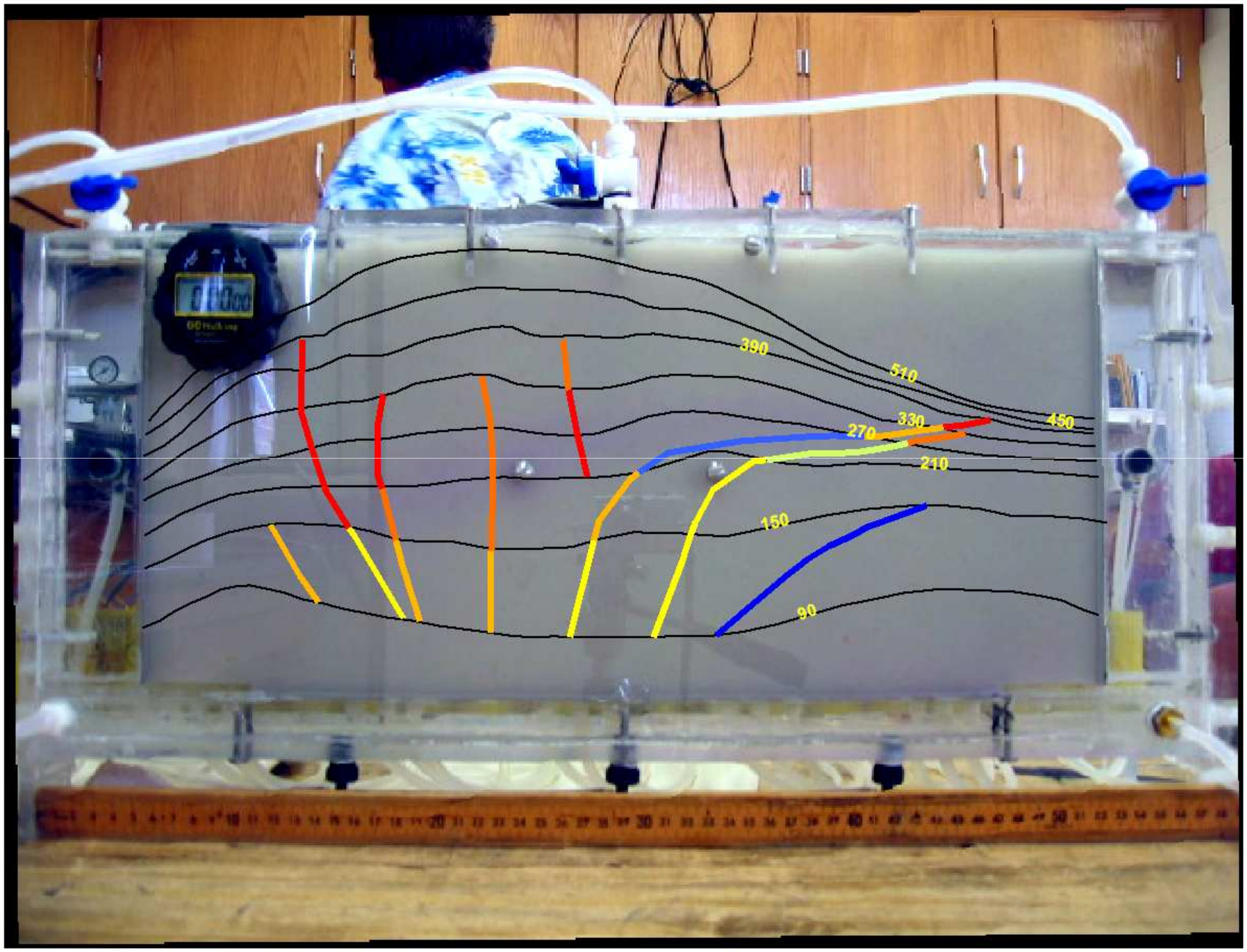
t=122s



Distribution of Tracer
0.25
0.2
0.15
0.1
0.05
0
z(m)
0
0.1
0.2
0.3
0.4
0.5
x(m)
1
0.8
0.6
0.4
0.2
0




510
390
330
450
270
210
150
90



Explanation

Velocity

cm/sec

- 0.027 - 0.040
- 0.041 - 0.060
- 0.061 - 0.080
- 0.081 - 0.100
- 0.101 - 0.120
- 0.121 - 0.140
- 0.141 - 0.160
- 0.161 - 0.180
- 0.181 - 0.200
- 0.201 - 0.220

CONCLUSIONS

- A new groundwater modeling approach is developed for groundwater flow in a karst aquifer having conduit and matrix regions. The Darcy system is used to describe flow in the matrix and stokes equation is adopted to describe the flows in conduits. The Beavers-Joseph interface conditions are applied at the interface between the two regions.

- The numerical simulation results for flow and solute transport match very well with laboratory experimental results. Thus, the developed mathematical and numerical models are physically verified and validated in the laboratory conditions.

- In comparison with CFP model, the Stokes-Darcy model doesn't require the exchange parameter, which is very difficult to obtain.

Acknowledgement

- This work is supported by the NSF CMG program under grant number DNS-0620035

Some Publications

Nan Chen, Max Gunzburger, Bill X. Hu, XiaomingWang, CelestineWoodruff, Calibrating the exchange coefficient in the modified coupled continuum pipe-flow model for flows in karst aquifers. Journal of Hydrology. Doi:10.1016/j.jconhyd2011.08.001, 2012.

Hu, B, X, Hua, F, M. Gunzburger, X. Wang, C. Yan, Experimental and computational validation and verification of the Stokes-Darcy and continuum pipe flow models for a karst aquifers with dual porosity structure, Hydrological Processes, DOI: 10.1127/hyp.8563, 2011.

Bill X. Hu**,** Examining a Coupled Continuum Pipe-Flow Model for Groundwater Flow and Solute Transport in a Karst aquifer, Acta Carsologica, 39/2, 347–359, 2010.

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Faulkner, J., B. X. Hu, S. Kish and F. Hua, Laboratory analog and Numerical study of groundwater flow and solute transport in a karst aquifer with conduit and matrix domains, Journal of Contamination Hydrology, 110, 34-44, 2009.

Acknowledgements

- Contributors to the research
  - Mr. Jonathan Faulkner – MS student (2009)
  - Dr. Fei Hua-Ph.D. student (2010)
  - Mr. Josue Gallegos, MS student (2011)
  - Prof. Max Gunzburger, Dept. of Computational Sci.
  - Prof. Xiaoming Wang, Dept. of Math
  - Prof. Yanzhao Cao, Dept. of Math, Auburn
  - Mr. Hal Davis, USGS
  - David Loper, FSU
  - Rodney DeHan, Florida Geological Survey

Questions?